

POLICY RESEARCH WORKING PAPER

Single-Equation Estimation of the Equilibrium Real Exchange Rate

John Baffes

Ibrahim A. Elbadawi

Stephen A. O'Connell

The World Bank
Development Research Group
August 1997

WPS1800

1800

An econometric methodology
for estimating both the
equilibrium real exchange
rate and the degree of
exchange-rate misalignment.



Summary findings

Estimating the degree of exchange-rate misalignment remains one of the most challenging empirical problems in an open economy. The basic problem is that the value of the real exchange rate is not observable.

Standard theory tells us, however, that the equilibrium real exchange rate is a function of observable macroeconomic variables and that the actual real exchange rate approaches the equilibrium rate over time.

A recent strand of the empirical literature exploits these observations to develop a single-equation approach to estimating the equilibrium real exchange rate.

Drawing on that earlier work, Baffes, Elbadawi, and O'Connell outline an econometric methodology for estimating both the equilibrium real exchange rate and the degree of exchange-rate misalignment.

They illustrate the methodology using annual data from Côte d'Ivoire and Burkina Faso.

This paper — a product of the Development Research Group — is part of a larger effort in the group to investigate the determinants of the real exchange rate. Copies of the paper are available free from the World Bank, 1818 H Street NW, Washington, DC 20433. Please contact Pauline Kokila, room N5-030, telephone 202-473-3716, fax 202-522-3564, Internet address pkokila@worldbank.org. August 1997. (51 pages)

The Policy Research Working Paper Series disseminates the findings of work in progress to encourage the exchange of ideas about development issues. An objective of the series is to get the findings out quickly, even if the presentations are less than fully polished. The papers carry the names of the authors and should be cited accordingly. The findings, interpretations, and conclusions expressed in this paper are entirely those of the authors. They do not necessarily represent the view of the World Bank, its Executive Directors, or the countries they represent.

Single-Equation Estimation of the Equilibrium Real Exchange Rate

**JOHN BAFFES
IBRAHIM A. ELBADAWI
STEPHEN A. O'CONNELL**

We are grateful to Chris Adam, Neil Ericsson, Philip Jefferson, and Luis Servén for helpful advice, to Peter Montiel for very thorough comments on an earlier draft, and to Ingrid Ivins for assistance with data. Larry Hinkle provided invaluable comments and advice throughout and constructed the counterfactual simulations for Côte d'Ivoire and Burkina Faso.

Contents

1. Introduction
 2. The equilibrium real exchange rate
 - 2.1 Rationing of foreign credit
 - 2.2 The terms of trade and trade policy
 - 2.3 Nominal rigidities and short-run dynamics
 - 2.4 Real exchange rate misalignment
 3. Estimating the equilibrium real exchange rate: introduction
 - 3.1 Specifying an empirical model
 - 3.2 Small samples, limited information, and the single-equation approach
 - 3.3 Sustainable fundamentals and exogeneity requirements
 - 3.4 Relationship to the PPP approach
 4. The econometric methodology
 - 4.1 Step 2: Estimation of
 - 4.1.1 The I(1) case: cointegration
 - 4.1.2 The I(1) case: estimation
 - 4.1.3 The I(0) case: estimation
 - 4.2 Step 3: Calculating the equilibrium real exchange rate
 - 4.2.1 Sustainable fundamentals: time-series-based estimates
 - 4.2.2 Sustainable fundamentals: counterfactual estimates
 - 4.2.3 Estimating the degree of misalignment
 5. Estimation results
 - 5.1 Unit root tests
 - 5.2 Tests of cointegration: Côte d'Ivoire
 - 5.3 Long-run parameters and adjustment speed: Côte d'Ivoire and Burkina Faso
 - 5.4 Short-run dynamics: Côte d'Ivoire and Burkina Faso
 - 5.5 The equilibrium real exchange rate and misalignment
 6. Conclusions
- Endnotes
- References
- Appendix 1: Conditioning and weak exogeneity
- Appendix 2: Data description
- Appendix 3: Counterfactual simulations: Côte d'Ivoire and Burkina Faso
 - A3.1 Time-series measures: TOT and LPFOR
 - A3.2 Counterfactual simulations: RESGDP

A3.3 Counterfactual simulations: ISHARE and OPEN1

A3.4 Concluding caveat

Figure 1: Internal and external balance

Figure 2: Adjustment to an increase in r_W (under a binding credit constraint)

Figure 3: Variance ratio tests for Côte d'Ivoire

Figure 4: Variance ration test for Burkina Faso

1. Introduction

Estimating the degree of exchange rate misalignment remains one of the most challenging empirical problems in open-economy macroeconomics (Edwards (1989), Williamson (1994), Hinkle and others (1995)). A fundamental difficulty is that the equilibrium value of the real exchange rate is not observable. Standard theory tells us, however, that the equilibrium real exchange rate is a function of observable macroeconomic variables, and that the actual real exchange rate approaches the equilibrium rate over time (Edwards (1989), Devarajan, Lewis and Robinson (1993), Montiel (1997)). A recent strand of the empirical literature exploits these observations to develop a single-equation approach to estimating the equilibrium real exchange rate (Edwards (1989), Elbadawi and O'Connell (1990), Elbadawi (1994), Elbadawi and Soto (1994, 1995)). Drawing on this earlier work, we outline an econometric methodology for estimating both the equilibrium real exchange rate and the degree of misalignment and illustrate the methodology using annual data from Côte d'Ivoire and Burkina Faso.

The procedure involves three steps. In the first step, the investigator examines the time-series characteristics of the real exchange rate and the fundamentals. This, in turn, determines the estimation technique to be used in the second step to uncover the parameters of the long-run relationship between the real exchange rate and its fundamentals. In the third step, the investigator uses the long-run parameters to calculate the equilibrium rate and the degree of misalignment under alternative assumptions regarding the sustainability of the fundamentals.

The paper is organized as follows. In section 2 we define the real exchange rate and derive the equilibrium relationship between the real exchange rate and macroeconomic "fundamentals" such as government spending patterns and the terms of trade. We present the comparative statics and discuss the sources of short-run misalignment and dynamic adjustment. Section 3 draws on the theory to develop a single-equation econometric model of the real exchange rate. In Section 4 we outline our methodology, and in Section 5 we apply the methodology to Côte d'Ivoire and Burkina Faso. Section 6 concludes with an assessment of the practical value of the single-equation econometric approach to the equilibrium real exchange rate.

2. The Equilibrium Real Exchange Rate

The concept of the real exchange rate (RER) that has been most heavily used in analyses of external

adjustment by developing countries is the domestic relative price of traded to nontraded goods (e.g., Dornbusch (1984)).¹

$$RER \equiv e \equiv \frac{EP_T^*}{P_N}$$

Although the foreign price of traded goods, P_T^* , is exogenous for a small country, the domestic price of nontraded goods is endogenous except over short periods of wage/price rigidity. The RER is therefore endogenous even under a predetermined nominal exchange rate. In this section we use a simplified model to illustrate the determination of the real exchange rate and derive an expression for its long-run equilibrium value. Since the relevant theory is well covered by Montiel, we use his model as a basis for the discussion (see also Edwards (1989) and Rodriguez (1994)).

The literature defines the long-run equilibrium real exchange rate as the rate that prevails when the economy is in internal and external balance for sustainable values of policy and exogenous variables. Internal balance holds when the markets for labor and nontraded goods clear. This occurs when

$$y_N(e) = c_N + g_N = (1 - \theta)ec + g_N, \quad y'_N < 0. \quad (2)$$

where y_N is the supply of nontraded goods under full employment, c is total private spending (measured in traded goods), θ is the share of this spending devoted to traded goods, and g_N is government spending on nontraded goods. Equation 2 is shown as the schedule IB in Figure 1. Starting in a position of internal balance, a rise in private spending creates an excess demand for nontraded goods at the original real exchange rate. Restoration of equilibrium requires a real appreciation that switches supply towards nontraded goods and demand towards traded goods. A rise in government spending on nontraded goods shifts the IB schedule downwards.

To define external balance, we begin with the current account surplus, which is given by

$$\dot{f} = b + z + rf = y_T(e) - g_T - (\theta + \tau)c + z + rf \quad (3)$$

where f is total net foreign assets, b is the trade balance, z is net foreign aid received by the government, and r is the real yield on foreign assets, measured in traded goods. The trade balance is the difference between domestic production of traded goods, y_T , and the sum of government (g_T) and private spending on these goods. The equation is standard except for the term τ which

measures the transactions costs associated with private spending. In Montiel's model of optimizing households, these costs motivate the holding of domestic money, which would otherwise be dominated in rate of return by foreign assets.² They are assumed to be incurred in the form of traded goods (at the rate τ per unit of spending) and therefore appear as an outflow in the trade balance.

External balance has been defined in various ways in the literature. The most useful approach for our purposes is that of Montiel (see also Khan and Lizondo (1987), Edwards (1989), and Rodriguez (1994)), who defines external balance as holding when the country's net creditor position in world financial markets has reached a steady state equilibrium. We can solve for the combinations of private spending and the real exchange rate that are consistent with this notion of external balance by holding f at its steady-state level and setting the right-hand side of equation (3) to zero. This traces out a second relationship between the real exchange rate and private spending, labeled EB in Figure 1. Starting at any point on this schedule, a rise in private spending generates a current account deficit at the original real exchange rate. To restore external balance, the real exchange rate must depreciate, switching demand towards nontraded goods and supply towards traded goods.

The equilibrium real exchange rate, e^* , is given by the intersection of the IB and EB curves, which occurs at point 1 in the diagram. Setting the right-hand-side of equation (3) to zero and combining this with equation (2), we obtain

$$e^* = e^*(g_N, g_T, r^* f^* + z, \tau^*), \quad e_1 < 0, e_2 > 0, e_3 < 0, e_4 > 0. \quad (4)$$

where "*" superscripts denote steady-state values of endogenous variables. The signs of the partial derivatives in (4) are easily verified either graphically or algebraically using equations (2) and (3).

Montiel solves for the steady-state service account $r^* f^*$ by assuming that the country faces an upward-sloping supply curve of net external funds and that households optimize over an infinite horizon.³ Transactions costs per unit, τ , are also endogenous; they depend on the ratio of money holdings to private spending and therefore on the nominal interest rate, which is the opportunity cost of holding domestic money. Since the nominal interest rate is tied down in the long run by the time preference rate and the domestic inflation rate, the final expression for the equilibrium real exchange rate in the Montiel model takes the form

$$e^* = e^*(g_N, g_T, z, r_w, \pi_T), \quad e_1 < 0, e_2 > 0, e_3 < 0, e_5 > 0. \quad (5)$$

where r_w is the world real interest rate and π_T is the rate of inflation in the domestic price of traded goods.⁴ Note that the nominal exchange rate does not appear among the fundamentals in equation (5). This is because the underlying behavioral relationships are all homogenous of degree zero in nominal variables. A nominal devaluation therefore has at most a transitory effect on the real exchange rate.

Equation (5) emphasizes that the real exchange rate consistent with internal and external balance is a function of a set of exogenous and policy variables. In practical applications, this relationship between e^* and its macroeconomic "fundamentals" differentiates the modern approach to equilibrium real exchange rates from the earlier PPP (Purchasing Power Parity) approach. Under PPP, the analyst would identify a reference period of internal and external balance and use the real exchange rate that prevailed during that period as an estimate of the equilibrium for other periods. Equation (5) implies that this is only legitimate if the fundamentals did not change between the reference and comparison periods. This criticism of the PPP approach is now widely accepted.⁵

The analysis underlying equation (5) can be readily modified to accommodate features that are important in particular applications. For our purposes, important extensions involve rationing of foreign credit, changes in the domestic relative price of traded goods, and short-run rigidities in domestic wages and prices. We discuss these extensions briefly in what follows.

2.1 *Rationing of foreign credit*

Equation (6) is derived under the assumption that the country faces an upward-sloping supply curve of external loans. The current account and trade balance are therefore endogenously determined at each moment by the saving and portfolio decisions of households. An extreme version of this view, more relevant for countries without access to commercial international borrowing on the margin, is that the country faces a binding credit ceiling (or equivalently, a floor on its international net creditor position). In this case, the trade surplus becomes exogenous, both in the short run and in the long run, provided that the credit ceiling remains binding.⁶ Equation (4) then takes the simpler form

$$e^* = e^*(g_N, g_T, b, \tau^*) \quad e_1 < 0, e_2 > 0, e_3 < 0, e_4 > 0. \quad (6)$$

In our empirical work below, we treat the trade surplus $b = rf + z$ as one of the fundamentals, consistent with this interpretation.

2.2 The terms of trade and trade policy

The domestic relative price of exports and imports is given by

$$\frac{P_X}{P_M} = \frac{\phi}{\eta}, \quad \phi \equiv \frac{P_{X^*}}{P_M^*}, \quad \eta \equiv \frac{1+t_M}{1-t_X}, \quad (7)$$

where ϕ is the external terms of trade and η is a parameter summarizing the stance of domestic trade policy. If either ϕ or η change over time, the analysis must be disaggregated to accommodate different real exchange rates for imports and exports. The equilibrium real exchange rates for imports and exports can then be written as functions of the set of fundamentals identified above, along with ϕ and η . Since the real exchange rate for tradables is itself a function of these two, it will depend on the same set of fundamentals, with elasticities depending on the relative weight (α) of imported goods in the tradables price index. Equation (6) then becomes

$$e^* = e^*(g_N, g_T, b, \phi, \eta, \tau^*), \quad e_1, e_3, e_\eta < 0; e_2, e_6 > 0; e_\phi ? \quad (8)$$

An improvement in the terms of trade increases national income measured in imported goods; this exerts a pure spending effect that raises the demand for all goods and appreciates the real exchange rate. This effect can be overcome by substitution effects on the demand and supply sides, leading to an overall real depreciation, but the spending effect has proved dominant in most empirical applications. A tightening of trade policy, appreciates the real exchange rate in the long run.

2.3 Nominal rigidities and short-run dynamics

In Montiel's model, domestic wages and prices are perfectly flexible and internal balance prevails continuously. If we consider the case of a binding credit ceiling, so that the trade balance is exogenous, we conclude that as long as changes in the fundamentals are permanent, the actual real exchange rate *never* deviates from its long-run equilibrium. This is apparent from inspection of the internal and external balance schedules: with b tied down exogenously, e and c are free to adjust immediately to their new long-run equilibrium values when one of the fundamentals changes. This is illustrated in Figure 2, where we show the adjustment to an increase in the world real interest rate by a net debtor country facing a binding credit ceiling. The rise in r_W increases the required trade surplus, shifting EB to the left (to EB') and depreciating the equilibrium real exchange rate. The

adjustment from point 1 to point 2 is immediate; with a predetermined path for the nominal exchange rate it takes place through a fall in domestic prices and wages. The binding credit constraint removes the model's only source of internal dynamics, so that the only possible sources of a divergence between the actual real exchange rate and its long-run equilibrium is a temporary change in one of the fundamentals.

If domestic wages and prices are sticky in the short run, a second important source of internal dynamics comes from disequilibrium in the labor market and the market for nontraded goods. As long as these markets eventually clear, the equilibrium real exchange rate is unaffected by the short-run nominal rigidity. But any shock that alters the equilibrium real exchange rate will now give rise to an adjustment process during which the actual real exchange rate will deviate from its new equilibrium. In Figure 2, sticky wages and prices prevent the real exchange rate from moving to point 2 in the short run, so that output and spending take the burden of the external adjustment. The short-run equilibrium is at point 3, where unemployment and inventory accumulation gradually push nominal wages and the prices of nontraded goods down relative to the prices of traded goods. The real exchange rate depreciates over time, bringing the economy to point 3 in the long run. The process illustrated in Figure 2 is often viewed as providing the primary role of nominal devaluation in macroeconomic adjustment (that of speeding an otherwise excessively slow and contractionary adjustment to an adverse external shock (Corden (1989))).

An advantage of the econometric methodology below is that it does not require a structural specification of the short-run dynamics. The long-run equilibrium is consistent with a variety of sources and patterns of short-run dynamics, including price stickiness, costs of labor mobility, and other features not present in the model above.

2.4 Real exchange rate misalignment

In the analysis below, we follow Edwards (1989) and Montiel in using the term "misalignment" to denote the gap between e and e^* . Two important differences between this descriptive use of the term misalignment and its more normative use in most policy discussions must be emphasized. The first is illustrated by our discussion of nominal rigidities. Without such rigidities, deviations between e and e^* are market-clearing responses to temporary movements in the fundamentals or to permanent movements that alter the long-run equilibrium level of net foreign assets. In these cases,

there is no obvious role for policy interventions designed to alter the path of the real exchange rate. The second difference stems from the observation that the real exchange rate may well be misaligned from a normative perspective even when the economic is in a steady-state equilibrium. Dollar (1993), for example, argues that African real exchange rates were systematically overvalued in the 1970s and 1980s, as a result of highly inward-looking trade regimes. In the theory developed here, the equilibrium real exchange rate is conditional on trade policies and other government interventions. Given these policy settings (whether socially optimal or not) misalignment is necessarily a temporary phenomenon, generated by short-run macroeconomic forces that prevent an immediate movement to the long-run equilibrium.

3. Estimating the equilibrium real exchange rate

The theory developed in the previous section delivers a steady-state, or long-run relationship between the real exchange rate and a set of macroeconomic “fundamentals.” The equilibrium real exchange rate is then defined as the steady-state real exchange rate conditional on a vector of permanent values for the fundamentals. Given this structure, our task is to construct a time series for the equilibrium real exchange rate – within sample and potentially out of sample – using data on the actual real exchange rate and fundamentals.

As a first step we assume that the long-run relationship delivered by theory is linear in simple transformations (e.g., logs) of the variables. Thus equation (5) becomes

$$\ln e_t^* = \beta' F_t^P, \quad (9)$$

where e^* is the equilibrium real exchange rate, F^P is the vector of permanent values for the fundamentals. At a conceptual level, the task of estimating the equilibrium real exchange rate breaks into two pieces. The first is to estimate the vector β of long-run “parameters of interest”; the second is to choose a set of “permanent” values for the fundamentals appropriate to period t .

3.1 Specifying an empirical model

Estimation of β requires the specification of an empirical model that is consistent with (9) but relates observable variables. We obtain such a model by translating into stochastic terms two straightforward and general features of the theory. The first is that equation (9) comes from a steady state relationship between *actual* values of the real exchange rate and fundamentals. To capture this

relationship we assume that the disturbance n_t in the equation

$$\ln e_t = \beta' F_t + n_t, \quad (10)$$

has finite conditional variance and expected value zero at sufficiently distant horizons (i.e., the limit of $E(n_{t+k}|I_{t-1})$ as k goes to infinity is 0).⁷ We will in fact impose the stronger condition that n_t is a mean zero, stationary random variable. Note that equation (9) follows directly from (10) if $\ln e^*$ and F^P are interpreted as long-run conditional expectations of the relevant variables.

The second general feature of the theory is that the steady state is dynamically stable.⁸ Shocks that cause the exchange rate to diverge from its (possibly new) equilibrium in the short run should produce eventual convergence to the relationship in (9) in the absence of new shocks (or equivalently, in conditional expectation). A specification that captures this notion while retaining consistency with both (9) and (10) is the general error-correction model

$$\Delta \ln e_t = \alpha(\ln e_{t-1} - \beta' F_{t-1}) + \sum_{j=1}^p \mu_j \Delta \ln e_{t-j} + \sum_{j=0}^p \gamma_j' \Delta F_{t-j} + v_t, \quad (11)$$

where $F_t = [g_N \ g_T \ b \ \phi \ \eta \ \tau^*]'$ is the vector of fundamentals, and v_t is an i.i.d., mean-zero, stationary random variable. Assuming that all variables are either stationary or $I(1)$ (see below), equation (11) implies equation (10); and for $\alpha < 0$, the corresponding long-run equilibrium is stable.

Equation (11) embodies the central insight of the single-equation approach: that the equilibrium real exchange rate can be identified econometrically as *that unobserved function of the fundamentals towards which the actual real exchange rate gravitates over time* (Kaminsky (1988), Elbadawi (1994), Elbadawi and Soto (1994, 1995)). Note that in contrast to the long-run relationship, the short-run dynamics are not heavily restricted since (11) is a simple re-parameterization of the unrestricted p^{th} -order autoregressive distributed lag (ADL) representation of $\ln e_t$,

$$\ln e_t = \sum_{j=1}^p \mu_j^* \ln e_{t-j} + \sum_{j=0}^p \gamma_j' F_{t-j} + v_t, \quad (12)$$

under the stability restriction $\sum_{j=1, p} \mu_j^* < 1$ and the assumption that the real exchange rate enters the long-run relationship.⁹ For different parameter values, the unrestricted error-correction

representation (11) encompasses a wide variety of commonly used dynamic models (Hendry, Pagan and Sargan (1984), Ericsson, Campos and Tran (1991)). This flexibility is an advantage, because although the dynamic structure of any *particular* theoretical model may place restrictions on the parameters in (9), these restrictions will depend on the nature of nominal and real rigidities, on whether households optimize or use rules of thumb, and on other model-dependent features that have little or no effect on the set of variables that enter the long-run equilibrium. With unrestricted dynamics, we allow the data maximum scope for determining their actual pattern, while retaining consistency with the long-run specification.

Much of our econometric work will take place in versions of equation (11). It is straightforward to incorporate variables that in theory do not belong among the long-run fundamentals, but that may affect the short-run dynamics; an example is the nominal exchange rate. Denoting such variables by z , we would capture long-term effects by adding the term $\delta'z$ inside the parentheses in (11) (allowing a test of the hypothesis $\delta = 0$) and short-term dynamics by adding $\sum_{j=0,p} \phi_j \Delta z_{t-j}$ to the right-hand side. Equation (11) can also accommodate an intercept or deterministic trend; and we can readily include dummy variables for potentially important exogenous events (e.g., the Sahel drought of the early 1980s).

3.2 *Small samples, limited information and the single-equation approach*

A fundamental difficulty in estimating the parameters of equation (11) is that sample sizes are likely to be very small. This is partly because the historical reach of developing country data is typically limited, and partly because models of the type considered here call for national accounts and/or fiscal data that are available only annually. For Côte d'Ivoire, we have 29 annual observations, and for Burkina Faso, 24. A general implication of small sample size is that the statistical properties of estimators may be poor and that testing procedures are likely to have low power. Existing Monte Carlo evidence can in some cases help discriminate between alternative choices of estimator, but we will often have to make informal judgments about robustness to sample size. On the positive side, the shocks to developing country data often appear to have high variance, thereby generating substantial variation over time; and temporal length of sample has the same effect when the real exchange rate and its fundamentals are nonstationary. A relatively small sample may therefore contain substantial information, particularly regarding the long-run parameter space.¹⁰

A second and more definitive effect of small samples in our case is to limit the scope for systems-based estimation. The number of unknown parameters in the full joint autoregressive distribution of the real exchange rate and its fundamentals rises roughly geometrically with the number of fundamentals and the lag length. With 3 or 4 variables among the fundamentals and fewer than 30 observations, this “curse of dimensionality” tends rapidly to overwhelm any attempt to estimate the full joint distribution. We will see below that the dimensionality problem is somewhat alleviated if the variables are nonstationary and cointegrated (and only the long-run parameters are of direct interest), but that even here the small sample size exerts a serious limitation on systems estimation. Our analysis will therefore generally take place in a single-equation context, where we implicitly condition on the current values of at least a subset of the fundamentals and the lagged values of all variables.

Conditioning is at some potential cost, because efficient statistical inference regarding the parameters of interest – which may go beyond β to include the adjustment speed α and the short-run parameters μ_i and γ_i – generally requires analysis of the full joint distribution of $\ln e_t$, F_t and z_t . As shown by Engle, Hendry and Richard (1983), however, fully efficient estimation and inference can take place conditional on the fundamentals if these variables are *weakly exogenous* for the parameters of interest. As outlined more fully in Appendix 1, weak exogeneity holds when the parameters of interest can be directly recovered from the distribution of the real exchange rate conditional on the fundamentals (and the past), *and* there are no cross-equation restrictions linking the parameters of this conditional model with those of the marginal model for the fundamentals. In this case the marginal distribution of the fundamentals holds no information of use to estimating the parameters of interest. Failure of weak exogeneity limits the scope for fully efficient conditional inference but may not undermine the ability to perform valid (though not fully efficient) inference in an essentially single-equation context; in the stationary case, for example, limited-information approaches like two-stage least squares are available subject to sufficient identifying restrictions.¹¹

For Côte d’Ivoire and Burkina Faso, the “small country” assumption suggests that variables like the terms of trade and the foreign price level are determined outside the country.¹² The same is true for the trade-weighted nominal exchange rate, since the CFA franc was pegged to the French franc at an unchanged parity throughout the sample; and the trade balance is in this category if borrowing constraints are exogenous and binding. Weak exogeneity seems a reasonable assumption

for these variables. Unfortunately, it is not guaranteed; if behavior is affected by conditional expectations of these variables, for example, forecast errors will be jointly determined with the real exchange rate, potentially violating weak exogeneity. Variables like government spending and the investment share may also be jointly determined with the contemporaneous real exchange rate. Weak exogeneity is testable, though generally at the cost of moving to systems estimation. Below we report some partial tests for the Côte d'Ivoire case.

3.3 *Sustainable fundamentals and exogeneity requirements*

If we begin with equation (10), the equilibrium real exchange rate in equation (9) has a natural interpretation as the limit of a k -period-ahead conditional forecast of the real exchange rate. This suggests two broadly alternative ways of tying down the permanent values of the fundamentals: the first is to use the sample information to generate long-run forecasts of the fundamentals conditional on information available in period t (or in some earlier period if t is out-of-sample); the second is to combine theory and *a priori* information into a counterfactual simulation for the fundamentals. These correspond closely to the use of a single equation for conditional forecasting and “policy analysis”. We argue below that the investigator will generally want to consider both alternatives. Here we briefly comment on the relevant exogeneity requirements (see Engle, Hendry and Richard (1983)).

The requirements for valid single-equation forecasting and simulation generally go beyond those for valid estimation and inference. When using conditional forecasts of the fundamentals, the implicit assumption is that there is no feedback from the real exchange rate to the fundamentals. The appropriate concept is *strong exogeneity*, which combines weak exogeneity with lack of Granger causality from the real exchange rate to the fundamentals. Given weak exogeneity, strong exogeneity can be readily tested by determining whether lagged values of the real exchange rate enter the marginal model for the fundamentals.

When using counterfactual simulations of the fundamentals, the relevant issue is whether β can be treated as a constant in the face of shifts in the marginal distribution of the fundamentals. The problem here is the Lucas critique of econometric policy analysis: the counterfactual exercise implicitly alters the joint distribution of the fundamentals and the real exchange rate, thereby invalidating the original parameter estimates unless the corresponding parameters are invariant to

the class of distributional shifts being considered. The appropriate concept in this case is *super exogeneity* which combines weak exogeneity with invariance of the parameters of interest to the class of distributional shifts under consideration. The invariance property is sensitive to the particular class of interventions under study and we will treat it as a maintained hypothesis rather than attempting formal testing.¹³

3.4 *Relationship to the PPP approach*

A hallmark of the PPP approach to equilibrium exchange rates was the choice of a single equilibrium rate for all periods, without reference to movements in the fundamentals. The standard theory-based criticism, as embodied in our theoretical model, was that notion of equilibrium delivers a *relationship* between the real exchange rate and the fundamentals, not a single value for the real exchange rate. Since the fundamentals are themselves time-varying, this criticism has often been summarized in the claim that the equilibrium real exchange rate should move over time.

The above discussion suggests, however, that this way of stating the criticism misses the fundamental distinction between the PPP and econometric approaches. Consider the case in which the real exchange rate itself is stationary. Stationary variables have time-invariant means, implying that all movements away from the mean are ultimately temporary. In such a situation the best sample-based estimate of the equilibrium real exchange rate for any period is simply the sample mean. To put this another way, the quantity $\beta'F_t$ in equation (10) is the difference between two stationary variables and is therefore stationary, so that while the individual fundamentals may have permanent movements (i.e., may be nonstationary), the relevant *function* of the fundamentals – in our case, the long-run forecast of a linear combination of these fundamentals – *never* moves permanently. When forecasted at successively distant horizons, $\beta'F_{t+k}$ simply reverts to the mean of $\ln e_t$.¹⁴ An equilibrium relationship between the real exchange rate and other macroeconomic variables is therefore consistent with a time-invariant equilibrium real exchange rate.

The more fundamental distinction between the two approaches resides in their contrasting use of sample and *a priori* information. The PPP approach requires a set of judgments that are informed both by theory and data but that remain largely implicit and *a priori* from an econometric perspective. The econometric approach, in contrast, uses theory sparingly but powerfully to extract information about the equilibrium real exchange rate from the entire data sample. *A priori*

information becomes relevant when the analyst is interested in counterfactual simulations for the fundamentals, but such information is combined with the sample information (used to estimate the parameters) in a restricted and transparent manner.

The econometric approach has clear advantages in reasonably large samples, where the high quality of the sample information should outweigh the loss of potentially sophisticated but implicit judgments central to the PPP approach. To give the PPP approach its due, however, we consider a problem that is peculiar to samples that are not necessarily small but are short in duration. We have just pointed out that in the stationary case, the sample mean provides a natural estimator of the long-run equilibrium real exchange rate. This implies, however, that the average misalignment within the sample is constrained to be zero. A similar though not identical outcome will tend to prevail in the nonstationary case: although the equilibrium rate itself is time-varying in this case, an important test of empirical success is that the equilibrium error is stationary. The resulting estimates of misalignment will then also tend to have a mean near zero if data-based forecasts for the fundamentals are used.

In other words, the econometric methodology tends by construction – except when counterfactual simulations of the fundamentals are used – to deliver an average misalignment of zero within the sample. This is in strong contrast to the PPP approach which embodies no such restriction. In large samples, the restriction of a near-zero average misalignment is an unambiguous virtue, since it imposes the structure required to uncover the long-run parameters. But there may be severe problems in small samples, particularly if adjustment speeds are slow. Côte d’Ivoire’s real exchange rate, for example, is thought by some to have been substantially overvalued for much of the post-WWII period. Our methodology, when applied using data-based permanent values for the fundamentals, is essentially incapable of reproducing this finding.

One response to this short-sample difficulty is to “re-base” the fitted equilibrium real exchange rates *ex post* by simply shifting their mean; this preserves their rates of change while altering the estimated degrees of misalignment. Despite its obvious appeal, however, rebasing has two important shortcomings. First, it leans very heavily on loosely structured *a priori* information, a feature of the PPP approach that the present approach is trying to avoid. Second, it embodies an implicit assumption of super-exogeneity with respect to potentially substantial and largely implicit interventions in the marginal distribution of the fundamentals. Our use of counterfactual simulations

for the individual fundamentals is a close cousin to the rebasing approach, but has the advantages of greater structure and transparency and, in particular, of exploiting the maintained super-exogeneity assumption more fully.

Viewed in this light, the PPP approach can be reinterpreted not primarily as an assumption that the equilibrium rate is a constant, but rather as an assumption that when samples are short and super exogeneity *fails*, loosely structured *a priori* information (e.g., “the economy was in internal and external balance in 1985”) is of greater value to the policy analyst than the information contained in the sample distribution of the real exchange rate and fundamentals, even when the latter is combined with structured *a priori* information about the fundamentals.

4. The econometric methodology

Given the structure just outlined, we suggest a three-step procedure for estimating the equilibrium real exchange rate. Step 1 is to determine the order of integration of the individual data series. Macroeconomic data often appear to possess a stochastic trend that can be removed by differencing once. Such variables are integrated of order one, or $I(1)$; they are nonstationary in levels and stationary after differencing. This pattern can readily be revealed using standard tests for the presence of a unit root. Other variables may prove stationary ($I(0)$) or trend-stationary (i.e., $I(0)$ after removing a deterministic trend component). The appropriate unit root tests are well known; in our applications we use the Dickey-Fuller (DF), augmented Dickey-Fuller (ADF), and Phillips-Perron (PP) tests. Although there are concerns about the low power of the unit root tests against stationary alternatives, the ADF test appears to perform satisfactorily on this score even when (as in our case) the number of observations is small (Hamilton (1994)). We also supplement the unit root tests with variance ratio tests (Cochrane (1988)) that exploit the fact that the variances of conditional forecasts explode for nonstationary series and converge for stationary series as the forecast horizon grows.

Steps 2 and 3 involve estimation of the long-run parameters and calculation of the equilibrium real exchange rate. Both steps are affected by the univariate time series properties of the data as revealed in step 1. In principle, the vector $[\ln e_t, w_t]'$ may contain an arbitrary combination of $I(0)$ and $I(1)$ (or even $I(2)$) variables. The examples studied here, however, fall into two extreme cases: in Côte d'Ivoire, we find that all variables are $I(1)$; in Burkina Faso, all variables are stationary in levels. We therefore restrict attention to these cases.¹⁵

4.1 Step 2: Estimation of β

When the variables are all $I(1)$, as in Côte d'Ivoire, stationarity of the residual n_t in equation (10) implies that the real exchange rate and its fundamentals are *cointegrated* (Granger (1981)). This property is extremely useful econometrically, and a massive literature has developed in the wake of Engle and Granger (1987).

4.1.1 The $I(1)$ case: cointegration

As shown by Johansen (1988), cointegration is a restriction on the reduced form or VAR representation of the joint distribution of the real exchange rate and its fundamentals. This reduced form can be written as

$$\Delta x_t = \Gamma x_{t-1} + \sum_{j=1}^p A_j \Delta x_{t-j} + \varepsilon_t, \quad (13)$$

where $x_t = [\ln e_t, F_t', z_t']'$ is the $nx1$ vector of variables and ε_t is the vector of reduced-form innovations (see Appendix 1). If the number of linearly independent stationary combinations of the variables is r ($0 \leq r < n$), then the matrix Γ in equation (A2) is of reduced rank $r < n$. We can then write $\Gamma = ab'$, where a and b are two nxr matrices of rank r .¹⁶ The columns of b span the “cointegrating space” of stationary combinations of the x_{it} ; the rows of a give the weights with which these combinations enter the individual equations of the reduced form. Equation (13) becomes

$$\Delta x_t = ab'x_{t-1} + \sum_{j=1}^p A_j \Delta x_{t-j} + \varepsilon_t, \quad (14)$$

Since the cointegrating vectors are identified only up to a normalization, we are free to impose r restrictions on the b matrix; for example, we might choose the normalization $b_{ii} = 1$, $i = 1, \dots, r$. We will restrict attention in this paper to the case in which $r = 1$, so that there is a single cointegrating vector. The normalization on $\ln e_{t-1}$ (which assumes only that $\ln e_{t-1}$ actually enters the long-run relationship) then exactly identifies the adjustment speed and the remaining components β_i of the cointegrating vector.¹⁷ With a single cointegration vector, then, a and b are $nx1$ vectors of the form $a = [a_1, a_2]$ and $b = [1, \beta]$.

We emphasized above that weak exogeneity is important both for fully efficient inference in a single-equation context and as a building block for the exogeneity concepts invoked in calculating the equilibrium real exchange rate. Urbain (1992) and Johansen (1992) show that the fundamentals are weakly exogenous for the long-run parameters and adjustment speed if the cointegration vector does not enter the marginal model for the fundamentals: $a_2 = 0$. This condition is equivalent to $\Gamma_2 = 0$ in equations (A3a) and (A3b) of Appendix 1; notice that it removes the cross-equation restriction that would otherwise have prevented the recovery of Γ_1 from the conditional model (A3a) alone.

Although we have been focusing on the long-run parameters, the investigator will typically be interested in the short-run dynamics as well. For example, policymakers confronting an overvaluation might want to know the short-run effects of a nominal devaluation. It is clear from equations (A3a) and (A3b) of Appendix 1 that the same condition that guarantees weak exogeneity with respect to the long-run parameters also guarantees weak exogeneity with respect to the short-run parameters *of the conditional model itself*. In reality, however, at least the long-run specification in (11) was derived not from conditioning but from a theoretical model. If we think of the short-run parameters of (11) as having similar structural interpretations, then the condition for weak exogeneity are more demanding. A set of sufficient condition is that $a_2 = 0$ and $\omega = 0$, where ω is the vector of covariances between the disturbance in equation (11) and the vector of disturbances from the marginal model (A3b) (Urbain (1992)).¹⁸

4.1.2 The $I(1)$ case: estimation

There are a number of potential approaches to estimating the cointegrating parameters. The simplest and earliest is the Engle-Granger (1987) “two-step” method, which applies OLS to a static regression relating the levels of the real exchange rate and its fundamentals (equation (10)). Cointegration implies that the residuals from this regression are stationary, and this restriction provides a test for cointegration. Because of the dominance of the common stochastic trend, the estimates of β from the static regression are super-consistent, approaching the true parameters at a rate proportional to the sample size rather than the square root of the sample size; and they remain so even in the absence of weak exogeneity. In the second step, lagged residuals from the static regression are used in place of the equilibrium errors on the right-hand side of a reduced-form error-correction equation. Again OLS provides consistent estimates, this time of the adjustment speed

and short-run parameters of the reduced-form error-correction specification.¹⁹

While the Engle-Granger method is extremely simple to implement, the estimates of the cointegration vector are biased in small samples. The degree of bias depends on the degree of persistence in the residual, suggesting that superior estimates might be obtained by accounting for the short-run dynamics (Banerjee, et al (1993)). We therefore also report OLS estimates of β taken directly from the error-correction specification (9). These control for the short-run dynamics – which may be of interest themselves – and, like the static regression estimates, remain consistent even with a failure of weak exogeneity.²⁰ Moreover, in line with our earlier discussion, a second and potentially decisive advantage emerges under weak exogeneity: estimates of β taken from the conditional error-correction model are equivalent to full-information maximum-likelihood estimates. They are therefore asymptotically efficient, and the t -ratios generated by OLS are asymptotically normal, allowing standard inference. This is in contrast to the static regression case, where the t -ratios have non-standard distributions even asymptotically.

A third natural alternative is the Johansen (1988) procedure, which is a systems approach based on estimation of the full VAR in equation (13). The “curse of dimensionality” is a serious limitation here, however. Monte Carlo evidence suggests that the Johansen procedure deteriorates dramatically in small samples, generating estimates with “fat tails” (i.e., frequent outliers) and sometimes substantial mean bias; moreover, it is less robust than the single-equation alternatives to misspecification of system parameters like lag length and to practical features like serial correlation in the equilibrium error (Hargreaves (1994)). Because of these small-sample limitations, we use the Johansen procedure to determine the number of cointegration vectors (i.e., to test for the rank of Γ in equation (13)) and to test for weak exogeneity – both of which are features of the entire system of equations – but otherwise restrict attention to the single-equation methods.

4.1.3 The $I(0)$ case: estimation

In the case of Burkina Faso, we find that all variables are stationary in levels. We pointed out above that in this case, the long-run “equilibrium” value of $\ln e_t$, like that of any stationary variable, is simply its mean. A consistent and efficient estimator of the equilibrium real exchange rate is therefore the sample mean, corrected for any deterministic trend. This implies that the long-run parameters need not be estimated for the purpose of tying down the long-run equilibrium. If the

fundamentals are super exogenous with respect to these parameters, however, a structural shift in the marginal process generating the fundamentals (for example, a shift in the mean of F_t) will produce a corresponding change in the mean of $\ln e_t$, with the slope of the effect given by the associated long-run parameter. Moreover, the long-run parameters and the short-run dynamics may be of theoretical interest even in the absence of super exogeneity; and the investigator may have a practical interest in generating short-to-medium-term conditional forecasts of the real exchange rate. For all of these reasons, we proceed with estimation in the stationary case even though it is not strictly necessary for assessment of the long-run equilibrium.

The theory of specification and estimation in the stationary case is well developed and we will not review it here; see Hendry (1995). What is clear is that the existence of a long-run relationship no longer exerts the kind of statistical leverage that it does when the variables are individually nonstationary. This is apparent in equation (10) since all the dynamics have been pushed into the residual n_t , which is therefore likely to be correlated with the right-hand side variables. OLS estimates of the static regression are therefore inconsistent in the $I(0)$ case, even though (as emphasized above) they are super-consistent when the variables are nonstationary and cointegrated.²¹ The error-correction model corrects this problem to some degree by incorporating dynamics; but the contemporaneous values of the fundamentals still raise issues of predeterminedness. Lacking identifying information on (11), one way to obtain consistent estimates of the parameters in that equation is to use higher lags of the fundamentals as instruments.²²

The lack of a clear statistical distinction between the individual and joint variation of the variables carries over to the conditions for weak exogeneity, which now make no general distinction between the short and long-run parameters. A sufficient condition in the present limited information context (i.e., where identifying restrictions on the marginal model are not available) is that equation (11) and the marginal model form a block-recursive system (which also obviates the need for instrumental variables and guarantees predeterminedness). We do not formally test for weak exogeneity in the $I(0)$ case (Burkina Faso), treating it instead as a maintained hypothesis where necessary (see Monfort and Rabemananjara (1990)).

4.2 Step 3: Calculating the equilibrium real exchange rate

Above we distinguished conditional forecasts and counterfactual simulations as two alternative

approaches to constructing sustainable values for the fundamentals. Here we broaden the first of these alternatives to consider various alternatives based on the time series behavior of the data. For policy purposes, concern often centers around the current or prospective situation rather than the historical episodes that make up the data sample. While our discussion focuses on within-sample estimates or simulations, the considerations outlined below apply equally to the construction of projected sustainable values for the fundamentals.

4.2.1 Sustainable fundamentals: time-series-based estimates

When the fundamentals are stationary, their movements are inherently temporary. We pointed out above that in this case the conditional long-run forecast is simply the sample mean (as corrected for any deterministic trend). At the other extreme all movements in the fundamentals are permanent. In this case, the fundamentals are individually random walks and the equilibrium real exchange rate in period t is simply F_t .

In practice, the fundamentals are likely to include both transitory and permanent components. This is clear for nonstationary fundamentals, where the permanent component corresponds to the underlying stochastic trend. The Beveridge-Nelson method, which we use below in the Côte d'Ivoire case, assumes that the fundamentals each follow a univariate ARIMA($p, 1, q$) process, with the autoregressive and moving average parts generating stationary fluctuations around an underlying random walk (Beveridge and Nelson (1981)). Movements generated by the unit root part are permanent and are extracted to construct F_t^P , the permanent component of F_t . The equilibrium rate is then given by F_t^P . This will tend to be a somewhat smoother series than F_t , reflecting the elimination of transitory shocks to the fundamentals.²³

We will also calculate sustainable values using centered moving averages of the fundamentals in both the stationary and nonstationary cases. This approach can be defended by appealing to the judgmental nature of the decomposition exercise and the disadvantages imposed by small samples. Moving averages mechanically smooth the data, to a greater degree the larger the number of periods used. In the nonstationary case, even a narrow moving average typically smooths the individual series more substantially than a B-N decomposition and may therefore yield results that are more appealing economically. The B-N approach is particularly problematic in small samples, where the results can be highly sensitive to the underlying ARIMA specification and can

often exacerbate turning points in economically implausible ways. This problem can affect the resulting equilibrium rate even more dramatically: if the fundamentals are all smoothed with a moving average, the resulting equilibrium rate is simply the corresponding moving average of F_t . The weighted sum of permanent components, in contrast, can easily be substantially more variable than F_t itself (as in our Côte d'Ivoire example below). Small samples also increase the possibility that stationary but persistent series are misidentified as nonstationary, in which case the B-N decomposition presumes a permanent component that in fact is not present.

In the stationary case, the moving average approach provides a way of acknowledging that even stationary fundamentals may have long-lasting movements. When a stationary variable is highly persistent, its conditional expectation at policy-relevant horizons can easily be relatively far from its unconditional mean. Using a moving average allows the long-run equilibrium rate to move in response to the current values of the fundamentals, even though these movements are thought ultimately to be temporary.

4.2.2 Sustainable fundamentals: counterfactual estimates

Ex-ante modeling of the permanent components of the fundamentals provides an important alternative to *ex-post* approaches that rely on the underlying data-generating processes of the fundamentals. There are two important reasons for pursuing this extension. The first is that in a small sample it may be virtually impossible, using time-series decomposition methods or moving averages, to distinguish persistent but unsustainable changes in the fundamentals from genuinely sustainable changes. The accumulation of international arrears by Côte d'Ivoire starting in the early 1980s provides an example: by this indicator, trade balances in that country appear to have been unsustainably large for over a decade. The second reason is that counterfactual simulations are needed to address the “what if” questions that are of central interest to policymakers, particularly when the fundamentals include variables potentially under policy control. Again using the Côte d'Ivoire case, policymakers might want to know the implications for the real exchange rate of a trade liberalization or change in government spending patterns. Preserving the relative simplicity of the single-equation approach, one way of handling these concerns is to construct counterfactual simulations of sustainable values for selected fundamentals. A potentially important side-effect of such simulations is to break the restriction implicit in the methodology that the average degree of

misalignment be near zero within sample.

In Appendix 2 we construct counterfactual simulations for the resource balance, openness, and investment share variables for both Côte d'Ivoire and Burkina Faso. For Côte d'Ivoire, these simulations incorporate the following judgments:

- the actual resource balance was unsustainably low after 1979.
- trade policy was unsustainably restrictive, particularly after 1979.
- the investment to GDP ratio was unsustainably low, particularly after 1979.

For Burkina Faso, the key judgments are (the investment to GDP ratio does not enter the model):

- the resource balance is determined by the volume of concessional inflows, and drought-year levels are unsustainable.
- trade policy was unsustainably restrictive throughout the sample.

Details of these calculations appear in Appendix 2.

4.2.3 *Estimating the degree of misalignment*

The estimated degree of misalignment, m , is simply the percentage difference between the real exchange rate and its computed equilibrium value:

$$m_t = \ln e_t = [\ln e_t - \beta' F_t^P] - \beta' (F_t - F_t^P) \quad (15)$$

For within-sample estimates, e_t is simply the actual real exchange rate. For out-of-sample estimates, e_t can be forecasted using a dynamic simulation that feeds projected paths for the fundamentals through the estimated short-run parameters of the model.

The degree of misalignment is decomposed mechanically in equation (15) into an error-correction term that captures the deviation of the exchange rate from the “fitted” real exchange rate using long-run parameters (the term in square brackets) and a term that captures the deviation of the current fundamentals from sustainable values. Expressing m this way brings out the role of sustainability calculations for the fundamentals. Suppose, for example, that the long-run parameter for the real exchange rate is negative, implying that a sustained terms of trade improvement appreciates the real exchange rate. If most movements in the terms of trade are temporary, however, and households optimize without borrowing constraints, then the short-run impact of a change in the terms of trade should be substantially below the estimated long-run impact (as in our theoretical model). A temporary improvement in the terms of trade would then produce offsetting changes in

the components of m_t : the second component would be large and negative, reflecting the temporary nature of the terms of trade boom; the first would be large and positive, reflecting the very modest response to the actual real exchange rate to the substantial short-run movement in F_t . Misalignment calculated using the actual rather than sustainable value of the terms of trade (i.e., setting $F_t^P = F_t$) would pick up only the second of these effects, producing the mistaken impression of a badly undervalued real exchange rate.

What the decomposition cannot do, is to identify the source of misalignment relative to plausible values for F^P . As discussed earlier, e_t may differ from F_t^P for reasons of real or nominal rigidities or, equivalently, equilibrium or disequilibrium dynamics; or it may be pushed by random shocks.

5. Estimation Results

In this section we illustrate the methodology using annual data for Côte d'Ivoire and Burkina Faso. We begin by noting that data limitations force two compromises in the estimation. The first is that we are unable to separate government spending into traded and nontraded goods. Data are available, however, for the shares of government consumption spending and total investment spending in potential GDP, and we use these to proxy for the level and composition of spending. A rise in government spending appreciates the real exchange rate if government spending is more intensive in domestic goods than is private spending; a rise in the investment share of GDP is likely to shift spending towards traded goods, other things equal, given the high import content of investment, and therefore to depreciate the real exchange rate. The second compromise is that in lieu of direct measures of the stance of trade policy, we must construct proxies for this variable. It is common in this literature to use various ratios of trade to GDP, on the argument that a more liberal trade regime, *ceteris paribus*, means higher trade volumes. We experimented with three such proxies: the ratio of current imports to current GDP, the ratio of constant-price imports to constant-price GDP, and the ratio of total trade (imports plus exports) to constant-price GDP. All three performed adequately for Burkina Faso, but in Côte d'Ivoire the ratio of current imports to current GDP was clearly superior to the other proxies. We therefore retain only this proxy in the analysis reported here. For the case of Côte d'Ivoire we also include a drought dummy variable that takes the value 1 for 1983 and 1984 and zero otherwise.

5.1 Unit Root Tests

As a first step we test all variables (except the drought dummy variable) for unit roots, to determine whether they can be represented more appropriately as stationary, difference-stationary, or trend-stationary processes. The results appear in Table 1. For Côte d'Ivoire, all three tests indicate nonstationarity for all variables. Moreover, we can reject the unit-root hypothesis for the first difference of the variables (not reported), so we conclude that these are $I(1)$ variables. For Burkina Faso, all variables appear to be trend stationary, with the possible exception of the terms of trade, which is bordering on nonstationarity. Figures 3 and 4 provide some additional information in the form of variance ratio tests.²⁴ These tests corroborate the unit root tests, and for Burkina Faso's terms of trade, the variance ratios decline at longer horizons, consistent with a persistent but stationary variable. We therefore proceed under the assumption that it is stationary.

Based on the above tests, we proceed to step two, in which we estimate the long- and short-run relationships between the real exchange rate and its fundamentals.

5.2 Tests of Cointegration: Côte d'Ivoire

Once it is confirmed that the variables behave as integrated processes, tests for cointegration can be undertaken to establish a long-run relationship between the RER and its fundamentals.²⁵ Table 2 reports the results of Johansen's likelihood ratio tests for the cointegrating rank of the system. We use a lag length of 1 for the underlying VAR system; this is very restrictive even for annual data, but longer lag length leaves us with very few degrees of freedom. The null hypothesis for these tests is that the number of cointegrating vectors relating the n nonstationary variables is less than or equal to r (where $r \leq n$). Comparing the estimated likelihood ratios in column 2 to the asymptotic critical values in column 3, we see (row 1) that the hypothesis of no cointegration ($r = 0$) can be rejected in favor of at most one cointegrating vector. In row 2, the hypothesis of one vector cannot be rejected in favor of more than one. The asymptotic tests therefore indicate one cointegrating vector.

Likelihood ratio tests of cointegration are known to be sensitive to small-sample bias, tending to reject low values of r too often. In column 4 we show a set of critical values that adjust for small-sample bias using a method suggested by Cheung and Lai (1993). Using these critical values it is difficult to distinguish between 0 and 1 cointegrating vector. We will proceed under the assumption that there is one vector, although we are marginally unable to reject the hypothesis of

zero at the 10% level using the adjusted critical values.

5.3 *Long-run parameters and adjustment speed: Côte d'Ivoire and Burkina Faso*

For Côte d'Ivoire we estimate the long run parameters using both the static regression, equation (10), and the error-correction model (ECM), equation (11).²⁶ We emphasized earlier the difficulties of system estimation in small samples, and the signs and magnitudes of the estimates obtained using the Johansen approach are less in line with the theory. We therefore confine attention to the results in Table 3. For Burkina Faso the long-run parameters are obtained from the ECM and appear in Table 6.

Columns 1 to 3 of Table 3 give static regression estimates for Côte d'Ivoire using alternative versions of the trade policy variable while column 4 shows the corresponding parameter estimates from an unrestricted ECM (recall that this is equivalent to the unrestricted ADL; the short-run parameters from this regression are presented in Table 4). As discussed earlier, both the static regression and the ECM deliver super-consistent estimates of long-run multipliers when cointegration is present. The estimates in columns 1 and 4 (which use the same openness variable) are reasonably close, providing some further support for the existence of a cointegrating relationship between the variables. In the static regression cases, we corroborate the existence of cointegration by applying unit-root tests to the estimated residuals (the critical values are more demanding than with a single variable, since the OLS estimation tends to induce stationarity in the residual); in each case the calculated values reject nonstationarity in favor of stationarity at standard levels. Since the OPEN1 results are generally strongest, we use this variable in what follows.

As discussed earlier, weak exogeneity holds with respect to the long-run parameters if the cointegrating vector does not enter the marginal model for the fundamentals. Engle and Granger (1987) suggest testing for weak exogeneity by introducing the error-correction term (the lagged residual from the static regression) into the equations of the marginal model and applying asymptotic *t*-tests to the hypothesis that the coefficients are zero. Using this test we are not able to reject weak exogeneity of the variables individually at reasonable significance levels, with the exception of ISHARE where we reject at the 5% level. Rejection for ISHARE suggests problems with inference in the error-correction specification: the long-run parameter estimates remain super-consistent, but standard errors are biased and inconsistent. To handle this we re-estimate the ECM

via instrumental variables, using lagged differences in the other fundamentals as instruments for ISHARE. These results appear in column 5 of Table 3 (the full ECM is column 3 of table 4). Inference can proceed from the IV version of the ECM, conditional on legitimacy of the chosen instruments.²⁷

For both countries, the estimated long-run parameters strongly corroborate the theoretical model. Estimated coefficients for the resource balance/GDP ratio are negative for both countries, as expected, suggesting that a rise in capital flows raises domestic absorption and shifts the composition of potential output towards nontraded goods. The implied elasticities of the real exchange rate with respect to the resource balance (-0.26 for Côte d'Ivoire and -1.50 for Burkina) are comparable in magnitude to those obtained in Elbadawi and Soto (1995) for Côte d'Ivoire and Mali.

The effects of shocks to the terms of trade (TOT), as remarked in Section 2, are theoretically ambiguous. However, consistent with the bulk of the empirical literature, the results here indicate that an improvement in the terms of trade appreciates the real exchange rate, suggesting that the spending effects of this variable dominate substitution effects. Again the terms of trade elasticity estimates are plausible as compared to other estimates in the literature. Perhaps the most interesting point is that despite the differences in the economic structure of the two countries, the extent of the effect appears to be the same for both countries (a 10% terms of trade improvement appreciates the real exchange rate by 4% in Côte d'Ivoire and by 3.5% in Burkina Faso).

In both countries a negative parameter estimate supports the notion that trade-liberalizing reforms are consistent with a more depreciated real exchange rate. The size of the elasticity differs: it is roughly -1 for Côte d'Ivoire and -0.47 for Burkina Faso. While these elasticities are not precisely estimated, they are consistent with evidence obtained by M'Bet and Madeleine (1995) and Elbadawi and Soto (1995) suggesting that the effects are stronger in the larger CFA countries.

For Côte d'Ivoire, a 10% increase in the share of investment in GDP (ISHARE) depreciates the real exchange rate by at least 2.7%, consistent with the view that this shifts the composition of spending toward traded goods. This evidence is consistent with that of Edwards (1989), but reveals an effect substantially lower than those estimates, which are in the range of 7% for a group of 12 LDCs. For Burkina Faso, the trend effect is significant, albeit with a very small coefficient, further corroborating the strong static trending effects revealed by the univariate unit-root tests.

To test the long-run homogeneity property – that the foreign price level, converted to CFA francs using the nominal exchange rate, does not affect the equilibrium real exchange rate – we included LPFOR in the specification and tested the null hypothesis of zero long-run coefficient. We use the dynamic regression results for this test since the t -statistics from the static regression have non-standard distributions even under weak exogeneity. For Burkina Faso, homogeneity cannot be rejected at any reasonable level of significance (Table 6). For Côte d'Ivoire, however, inclusion of the change in LPFOR (or just the change in the nominal exchange rate) in the ECM, causes marked deterioration of the results. Thus, while long-run homogeneity cannot be rejected, the remaining results are unsatisfactory. For the purposes of subsequent calculations, we treat long-run homogeneity as a maintained hypothesis.

5.4 *Short-run Dynamics: Côte d'Ivoire and Burkina Faso*

Tables 4 and 6 show the short-run parameters from the estimated ECMs for Côte d'Ivoire and Burkina Faso. For Côte d'Ivoire (Table 4) we show two alternatives, corresponding to the long-run parameters in columns 1 and 4 of Table 3. Column 1 uses the lagged residual from the static regression in column 1 of Table 3, so that the short-run parameters are estimated conditional on the cointegration vector from the static regression.²⁸ In column 2 we estimate the short-run parameters jointly with the long-run parameters using the unrestricted ECM. Columns 3 and 4 estimate these regressions using lagged differences of the other right-hand-side variables as instruments for ISHARE.

For the case of Côte d'Ivoire the short-run effects of the fundamentals are generally appreciable in size, statistically significant, and in the same direction as the long-run effects. For the case of Burkina Faso, the short-run impact effects are generally less than half the size of their corresponding long-run coefficients, and in most cases are statistically insignificant.

The short-run estimates provide direct evidence on the short-run effects of nominal devaluations on the real exchange rate. Under long-run homogeneity, nominal devaluations have at most a transitional role to play in macroeconomic adjustment. If domestic wages and prices are sticky, however, shocks that depreciate the equilibrium real exchange rate (e.g., a fall in sustainable capital flows, a persistent deterioration in the terms of trade deterioration, or a trade liberalization) will generate a contractionary adjustment under fixed exchange rates. Nominal devaluation can then

be an important part of efficient macroeconomic adjustment (e.g., Corden (1989)), provided that their short-run effects on the real exchange rate are not neutralized by domestic inflation. The results of the error-correction model corroborate this view for both countries. The estimated short-run elasticities are statistically significant, fairly substantial, and virtually identical across the two countries: these results suggest that a 50% devaluation (such as the one effected in 1994) will depreciate the real exchange rate by 15% in the short-run.²⁹

A crucial parameter in the estimation of these short-run dynamic models is the coefficient of the error-correction term which measures the speed of adjustment of the RER to its equilibrium level. The adjustment speed estimated for Côte d'Ivoire in Table 4 is lower (at -0.30 and -0.45, respectively, in the unrestricted and 2-step ECM) than the corresponding estimate for Burkina Faso in Table 6 (at -0.54). The adjustment speed for Côte d'Ivoire is somewhat higher than that obtained for Côte d'Ivoire by Elbadawi and Soto (1995) using a similar framework. From these estimates the number of years required to eliminate a given misalignment can be derived.³⁰ For example, eliminating 95% of a shock to the real exchange rate would take slightly more than 3 years in Burkina Faso and could take as long as 8 years in Côte d'Ivoire. Elbadawi and Soto (1995) find a similar difference in adjustment speed for Mali. This finding suggests that smaller economies in the zone are more adaptive to shocks than the larger ones. This conclusion appears consistent with the widely held view that the latter group of countries has experienced much higher degree of overvaluation during the 1986-94 period than the former one. One reason for this may be a greater prevalence of nominal rigidities in the larger economies, with their larger manufacturing sectors and a greater proportion of production for the domestic market.

Adjustment speeds for both countries, however, are substantially larger than the -0.19 obtained by Edwards (1989) for a group of developing countries using a partial adjustment model. To the degree that these adjustment speeds can be legitimately compared, they may provide some support for the view that membership in a monetary union produces greater flexibility of nominal wage settlements in the private sector by increasing the credibility of monetary policy (Rodrik (1994)).

5.5 The equilibrium real exchange rate and misalignment

Tables 5 and 7 show alternative measures of the equilibrium real exchange rate. For Côte d'Ivoire,

we use the long-run parameters derived from the static regression in column (1) of Table 3. For Burkina Faso, we use the long-run parameters from the unrestricted ECM in column 1 of Table 6.

For Côte d'Ivoire we report four measures of the “equilibrium” real exchange rate: the fitted RER, its corresponding 5-year moving average, an equilibrium rate based on Beveridge-Nelson decompositions of the fundamentals, and one based on the counterfactual simulations (Appendix 3). For Burkina Faso, we replace the B-N decomposition with the fitted trend for the real exchange rate; as discussed earlier, this represents the most natural long-run forecast for a trend-stationary variable.

Recall that when we generate long-run “forecasts” of the real exchange rate using time-series-based estimates of the “permanent” fundamentals, we require not only adequate estimates of the long-run parameters but also a lack of Granger causality from the real exchange rate to the fundamentals. With a lag length of 1, weak and strong exogeneity coincide and the partial tests reported earlier for Côte d'Ivoire therefore provide some support for these calculations. As an additional check, we tested the multivariate generalization of Granger non-causality from the real exchange rate to the fundamentals and were unable to reject non-causality at any reasonable levels, using a lag length of either 1 or 2. As argued earlier, the use of counterfactual simulations for the fundamentals involves an assumption that the long-run parameters are invariant to the interventions being constructed; we treat this as a maintained hypothesis.

The last columns of Tables 5 and 7 show the percentage gap between the observed and equilibrium real exchange rates, using the counterfactual simulations for the equilibrium rate. The gap between these two series provides a measure of real exchange rate misalignment. The figures show a remarkable success on the part of the computed index in reproducing well-known overvaluation (and undervaluation) episodes in the recent macroeconomic history of these countries and the CFA zone more generally. In particular, note that Côte d'Ivoire managed to reverse substantial real overvaluation by the end of the first half of the 1980s. While some of this was generated by contractionary macroeconomic policies that fell heavily on investment, a substantial contribution came from the steady depreciation of the French franc against the US dollar and other major currencies. When the French franc moved in the reverse direction following 1986, the fiscal laxity and structural rigidities that characterized the Ivoirian economy all along were fully exposed; our calculations imply that during the 1987-93 period the real exchange rate was overvalued by 34% on average. By 1994 a set of corrective measures, including a zone-wide 50% devaluation, were

effected. Using a CGE approach, Devarajan (1997) estimates the degree of overvaluation in Côte d'Ivoire for 1993 at 36%, a figure close to our estimates based on counterfactual simulations of the fundamentals.

For Burkina Faso, in contrast, our results for 1980-93 do not indicate any major overvaluation. Indeed, according to our estimates, Burkina Faso's real exchange rate was undervalued by 1% on average between 1980 and 86 and by nearly 14% during 1987-93. The estimated undervaluation may be on the high side for the latter period.³¹ Burkina Faso is generally regarded, however, as having adjusted successfully to the adverse shocks that affected the entire zone after 1986, especially in comparison with the larger CFA countries (Devarajan and Hinkle (1995)). Substantially milder overvaluation (or even undervaluation) is one measure of this success; another is the absence in Burkina Faso of the deep recession experienced by Côte d'Ivoire experienced during the 1980s and early 1990s. Both observations suggest a more flexible domestic wage and price structure in the smaller of the two countries, and therefore significantly milder nominal rigidities.

Conclusions

The decision to devalue depends fundamentally on degree of misalignment of the real exchange rate and the speed with which internal adjustment mechanisms are likely to restore macroeconomic balance. Measuring the degree of misalignment is difficult, however, given that the equilibrium real exchange rate is unobservable whenever the economy is not in internal and external balance. The standard PPP approach is to identify a period in which the economy is judged to have been in balance, and to take the real exchange rate of that period as the equilibrium rate for all years. But this fails to account for the effect of changes in the fundamentals on the equilibrium real exchange rate.

Once the endogeneity of the equilibrium real exchange rate is recognized, however, a second problem arises: restricting attention to plausible candidates for years of macroeconomic balance, there will rarely be enough observations to estimate the elasticities of the equilibrium rate with respect to even a small list of fundamentals. In this paper, we addressed these problems by imposing the relatively mild (and testable) restriction, drawn from standard theories of the equilibrium real exchange rate, that the distance between the actual and equilibrium real exchange rates is a

stationary random variable. When the variables are $I(1)$, this leads naturally to the use of cointegration methods for estimating the long-run relationship between the real exchange rate and its fundamentals. When the variables are stationary, standard procedures of dynamic specification and estimation apply. We illustrated the methodology using annual data for Côte d'Ivoire and Burkina Faso.

How useful an addition is this methodology to the standard toolbox for assessing the equilibrium real exchange rate and the degree of misalignment? Our view is that this methodology belongs in the analyst's toolkit as a clear advance over PPP and a useful complement to other methods. There are three fundamental reasons for this.

- First and foremost, this approach provides a natural way of incorporating the reality that the fundamentals will sometimes move permanently. In such a case our approach extracts maximal leverage from the theoretical proposition that the real exchange rate will not stray indefinitely from a function of the fundamentals.
- Second, estimating the equilibrium real exchange rate is typically motivated by policy concerns. The analyst may therefore be particularly interested in the relationship between the equilibrium real exchange rate and hypothetical changes in individual fundamentals. For out-of-sample exercises, interest would center on how changes in the fundamentals would alter both the actual and the equilibrium rates, and thereby the degree of misalignment. Under super exogeneity of the fundamentals, our approach delivers a set of parameters that can be used for such policy analysis in a transparent and straightforward manner.
- Third, this approach takes a partial step towards imbedding the determination of the long-run relationship in the overall dynamic relationship between the real exchange rate and its fundamentals. Under weak exogeneity with respect to the short-run parameters, fully efficient estimation and inference on these parameters can take place conditional on the current and lagged fundamentals. The resulting information on short-run dynamics may be of interest in its own right and if Granger non-causality also holds, can be used to generate short-term forecasts of the real exchange rate and degree of misalignment.

From the general viewpoint of dynamic specification, there are various directions in which the approach advocated here might be extended. One is to allow both $I(0)$ and $I(1)$ variables in the long-run relationship. In this case the theory still implies cointegration among the $I(1)$ variables, but the Engle-Granger two-step method will produce inconsistent estimates of the long-run parameters on the stationary variables. We are therefore pushed towards allowing explicitly for the short-run dynamics, whether via the error-correction model, the Johansen procedure, or some alternative. A

second extension would be to allow multiple long-run relationships between the variables. Such a case might arise, for example, from the existence of a reaction function relating fiscal policy to the trade balance and/or the real exchange rate; moreover, since most of our variables are already measured in ratios (the real exchange rate, the openness variable, etc.), we may already be reducing the order of integration of underlying nonstationary variables (like the domestic price level). The structural error-correction model of Boswijk (1995) represents the closest counterpart to our analysis in the case of multiple cointegrating relationships. Finally, we have chosen not to impose any theoretical structure on the short-run dynamics. In cases where particular sources of short-run dynamics are of interest, there may be a substantial return to developing a theoretical structure to capture these dynamics, and imposing the resulting identifying restrictions; for an interesting attempt to incorporate rigidity of domestic prices, see Kaminsky (1988). An interesting challenge along these lines is that of separating misalignments due to price rigidities from those due to internal real dynamics or temporary movements in the fundamentals.

Naturally, most of these extensions will bring out a tension between maintaining the simplicity of a single-equation approach – an important feature if this approach is to be used intensively “in the field” – and allowing the overall dynamic relationship(s) to emerge from the data. Here we identify three particular cautions regarding the use of this methodology in the policy arena.

First, the econometric approach is data-intensive and inherits all the limitations of developing country data. Our empirical findings for Côte d'Ivoire and Burkina Faso are broadly consistent with the empirical literature on equilibrium real exchange rates in developing countries, and they line up well with estimates obtained by other methods. But they are not robust. We noted above that the econometric results were quite sensitive to the choice of proxies for the fundamentals and to the estimation procedure. The choice of real exchange rate index also made a difference empirically, and although changes in long-run elasticities are to be expected, we found that the overall statistical performance was highly sensitive to whether an internal or external real exchange rate concept was employed (see Hinkle et al.) and whether the nominal exchange rate was adjusted for black market transactions. While such conditions define the art of econometric practice, they may be particularly acute when the notion of long-run equilibrium in short samples is required to carry so much weight in short samples.

The reality of short samples brings out a second potential weakness in this approach, even relative to the PPP approach. In effect, the single-equation methodology assumes that the economy was in internal and external balance *on average* over the sample period. This avoids the need for *a priori* and possibly *ad hoc* claims about macroeconomic balance in any particular year, providing instead a systematic way of bringing the whole sample to bear in determining the path of the equilibrium rate. But it implies that unless the analyst is prepared to undertake counterfactual simulations for the fundamentals, the average degree of misalignment in the sample will tend, by construction, to be small. There will be little scope for uncovering episodes of over- or under-valuation that last more than a few years. In the CFA zone, where the real exchange rate was widely thought to be overvalued for most of the period between 1978 and 1994, the implicit “balance on average” assumption may be seriously misleading. The PPP approach, of course, does not require such an assumption; the result is that the real exchange rate can in principle be overvalued (or undervalued) in every other period than the benchmark one. We suggested that a natural way of handling this within our methodology was to construct counterfactual simulations for the fundamentals. In our counterfactuals for Côte d'Ivoire, freer trade, higher domestic investment, and smaller trade deficits all produced a depreciation of the equilibrium rate and therefore tended to increase the estimated degree of misalignment.

Finally, the methodology relies on concepts of equilibrium and misalignment that are conditional on policies or structural features that can reasonably be treated as predetermined, whether or not those policies or features generate welfare losses. In this view, short-run misalignments may well reflect market-clearing responses to shocks, and long-run movements in the real exchange rate may well reflect highly suboptimal macroeconomic policy choices. For both reasons the misalignments most readily identified using single-equation time-series methods may not be the most interesting from a policy perspective.

Endnotes

1. This is what Hinkle et al. call the "internal" real exchange rate.
2. Montiel assumes that transactions costs are a decreasing function of the ratio of money holdings to spending: $\tau = \tau(m/c)$, $\tau' < 0$.
3. The latter feature ties the domestic real interest rate to the time-preference rate in any steady state. Given r^* , the value of f^* is then determined uniquely by the external supply function.
4. Since $p_T = p_W + e$, where p_W is the world inflation rate, we can think of the world rate of inflation and the domestic rate of crawl of the exchange rate as among the fundamentals. Also, we have suppressed the time preference rate in writing equation (5).
5. Williamson (1994), for example, writes that "PPP comparisons are indispensable for comparing living standards, but they are the wrong basis on which to calculate equilibrium real exchange rates. They are wrong conceptually, and they provide seriously misleading advice. For that purpose they should be abandoned, once and for all." (p. 191). In section 3 below we discuss further the distinction between the PPP approach and our approach.
6. The domestic real interest rate, in contrast, becomes endogenous. Movements in the domestic real interest rate reconcile private spending decisions with the exogenous credit constraint, with the spread between the domestic and foreign real interest rates capturing the shadow price of the credit constraint.
7. By finite conditional variance we mean that $Var(n_{t+k}|I_{t-1})$ is finite, where the information set I_{t-1} contains all values of $\ln e$ and F dated $t-1$ or earlier.
8. This does not rule out theoretical models that exhibit instability in certain directions (e.g., rational expectations models); the key assumption is that the economy "chooses" a convergent path for given values of the fundamentals.
9. In terms of the ADL parameters, the adjustment speed and long-run parameters β_i in the error-correction representation are given by $\alpha = (\sum_{j=1,p} \mu_j^*)$ (so that the stability restriction implies $\alpha < 0$) and $\beta_i' = -(\sum_{j=1,p} \gamma_j')/\alpha$.
10. In the end, this high variability is useful only if it can be parameterized in a sufficiently parsimonious manner. With so few data we are virtually forced into assuming that the parameters are constant over the sample. This rules out structural changes that may in fact be important over the sample period and, if incorrect, can produce misleading inferences about the nonstationarity of the data and the values of the parameters.
11. In the stationary case, specifying a dynamic simultaneous model or even a just-identified structural VAR' along the lines of Bernanke (1986) would require more identifying information than we are willing to impose *ex ante*. Moreover, systems-based estimates that exploit this information are known to be less robust to misspecification than limited-information approaches that ignore identifying information outside of the equation being estimated.
12. Côte d'Ivoire may well be large enough in the world cocoa market to affect its terms of trade.

13. See Hendry (1995). Not surprisingly, tests for super exogeneity generally require intensive study of the relationship between the estimated equation and the associated reduced form for the fundamentals. One natural test (given weak exogeneity) is to establish parameter constancy in the estimated model given a sample break in the associated reduced form for the fundamentals.
14. Estimation of long-run parameters appears superfluous in this case. The investigator will typically be interested, however, not only in a good conditional forecast of the real exchange rate, but also in various characteristics of the short-run dynamics. Uncovering the relevant parameter estimates requires estimation even in the stationary case. Moreover, estimates of β are also required to apply counterfactual simulations for the fundamentals.
15. Methods have recently been developed that allow consistent estimation and inference in regressions that involve mixtures of integrated processes; see Phillips (1995) and Phillips and Chang (1995).
16. Since each of the variables in x is either $I(0)$ or $I(1)$, all of the first differences in (14) are stationary. Stationarity of ε_t then implies that each row of Γx_{t-1} must also be stationary (since it is a linear combination of stationary variables) although the individual x_{it} 's are all nonstationary. This is accomplished if the rows of Γ induce linear combinations of stationary linear combinations of the nonstationary variables x_t ; hence the decomposition of Γ . Note that if there are n stationary combinations, then the individual x_{it} 's must all be stationary.
17. If there are multiple cointegrating relationships, normalization alone is insufficient to relate the long-run parameters uniquely to their counterparts in any particular economic theory – i.e., to obtain “interpretable” parameter estimates – and we require further identifying restrictions (see, for example, Johansen and Juselius (1994)). In this case the single-equation approach is likely to pick up a weighted combination of the cointegration vectors (Johansen, 1992). This lack of identification may not be highly damaging for forecasting purposes, but it raises a variety of issues that go beyond the scope of single-equation approaches. The closest counterpart to our approach in the $r > 1$ case is the “structural error correction model” of Boswijk (1995) (discussed in Ericsson (1995)), which is obtained by premultiplying (14) by a square matrix and then imposing a set of restrictions.
18. Under these conditions case equation (11) and the unrestricted reduced forms (A3b) form a block-recursive system.
19. Engle and Granger (1987) demonstrated an equivalence between cointegration and error correction for nonstationary variables. In the nonstationary case, therefore, equation (10), which implies cointegration, also implies that the real exchange rate has a reduced-form error-correction representation, i.e., one that is similar to (11) but with contemporaneous values of the fundamentals excluded. It is this reduced-form error-correction equation that is estimated in the second step of the Engle-Granger method.
20. A failure of weak exogeneity, however, means small-sample bias and invalid inference regarding the long-run parameters. Recall also that the conditions for weak exogeneity with respect to short-run parameters are stronger.
21. The standard sufficient condition for consistency of OLS in the stationary case is that the right-hand side variables are *predetermined*, i.e., that the residual is uncorrelated with contemporaneous and lagged right-hand side variables. In equation (10) the condition is $Cov(n_t, x_{t-k}) = Cov(n_t, w_t) = 0$. In the stationary case, predeterminedness corresponds closely (but not exactly) to weak exogeneity (Engle, Hendry and Richard (1983), Monfort and Rabemananjara (1990)).

22. See Monfort and Rabemanajara (1990) for development of exogeneity concepts and tests in the stationary context.
23. Any set of cointegrated variables has a common trend representation; this could be the basis of a joint decomposition of the real exchange rate and fundamentals into a stochastic trend component and a stationary (moving average) component (see Banerjee, et al (1993)). The B-N approach approximates this by treating the variables one by one.
24. This ratio is defined as $(1/k)var(X_t - X_{t-k})/Var(X_t - X_{t-1})$, where X_t is the variable of interest and k is the lag length (Cochrane, 1988).
25. We include the drought variable in the long-run relationship, on the grounds that it picks up a supply shock that is highly asymmetric between traded and nontraded goods. Unfortunately, the critical values of Dickey-Fuller tests and the many of the tests used in the Johansen procedure are sensitive to the exact specification of deterministic variables in the cointegrating relationship. We do not attempt the Monte Carlo simulations that would be required to establish critical values for our case.
26. We apply instrumental variables (IV) to the Bewley transform of equation (14), using the ADL variables as instruments. This gives numerically equivalent results to using OLS on the ADL representation. The advantage of the Bewley transform is that the long-run parameters and their standard errors can be read directly from the equation. See Banerjee, et al, pp. 55-64.
27. Although these results are encouraging, weak exogeneity may be a more serious problem than is indicated by our variable-by-variable tests. Using Johansen's system-based chi-squared test, we strongly reject joint weak exogeneity for the fundamentals taken together.
28. Note that this is not the same as the error-correction representation referred to in the Granger Representation Theorem (Engle and Granger, 1987). The latter is a reduced-form equation that omits contemporaneous changes of the fundamentals.
29. The calculation for Côte d'Ivoire relies on the second-stage ECM estimates. As discussed earlier, the dynamic regression estimates are unsatisfactory when LPFOR is included.
30. The time required to dissipate $x\%$ of a shock is determined according to: $(1-\beta)^t = (1-x)$, where t is the number of years and β is the absolute value of the speed of adjustment parameter.
31. For example Elbadawi and Soto (1995), using a similar methodology, estimate that the RER in Mali was virtually in equilibrium (on average) during the 1987-94 period, while the CGE estimates of Devarajan (1997) suggest that the RER in Burkina Faso was overvalued by about 9% in 1993.

TABLE 1: Stationarity Statistics – Levels without and with Time Trend

	<i>Côte d'Ivoire</i>			<i>Burkina Faso</i>		
	<i>DF</i>	<i>ADF</i>	<i>PP</i>	<i>DF</i>	<i>ADF</i>	<i>PP</i>
<i>Levels without Time Trend</i>						
<i>log(REER)</i>	-0.59	-1.26	-1.89	-2.25	-4.25	-2.25
<i>log(TOT)</i>	-1.42	-1.54	-1.78	-1.95	-1.82	-1.87
<i>RESGDP</i>	-2.11	-2.57	-2.25	-3.84	-2.22	-4.07
<i>log(OPEN1)</i>	-1.06	-1.39	-1.42	-4.02	-3.04	-4.30
<i>log(OPEN2)</i>	-2.35	-1.99	-2.48	-3.23	-3.02	-3.35
<i>log(OPEN3)</i>	-2.52	-2.16	-2.69	-3.63	-2.99	-3.82
<i>log(ISHARE)</i>	-1.01	-0.78	-0.68			
<i>Levels with Time Trend</i>						
<i>log(REER)</i>	-1.83	-2.46	-2.09	-4.89	-2.76	-5.35
<i>log(TOT)</i>	-1.51	-1.56	-1.69	-2.30	-2.08	-2.34
<i>RESGDP</i>	-2.05	-2.50	-2.24	-4.27	-2.69	-4.64
<i>log(OPEN1)</i>	-1.02	-1.32	-1.29	-3.84	-2.94	-4.20
<i>log(OPEN2)</i>	-2.81	-2.30	-3.02	-3.12	-2.95	-3.31
<i>log(OPEN3)</i>	-2.47	-1.99	-2.72	-3.47	-2.91	-3.75
<i>log(ISHARE)</i>	-2.42	-2.19	-2.42			

NOTES: DF, ADF, and PP refer to Dickey-Fuller, augmented Dickey-Fuller, and Phillips-Perron stationarity statistics. The number of observations is 29 for Côte d'Ivoire and 24 for Burkina Faso. The variables are defined in Appendix 2 (ISHARE is not available for Burkina Faso).

Table 2: Johansen's Maximum Likelihood Test of Cointegration Rank for Côte d'Ivoire

	L-Max	10% critical value		5% critical value	
		unadjusted	adjusted	unadjusted	adjusted
With the dummy					
$r = 0$	45.01	36.35	48.34	39.43	52.44
$r \leq 1$	30.05	30.84	41.02	33.32	44.31
Without the dummy					
$r = 0$	32.65	30.84	39.17	33.32	42.32
$r \leq 1$	18.63	24.78	31.47	27.14	34.47

NOTES: The first row ($r = 0$) tests the null hypothesis of no cointegration; the second ($r \leq 1$) tests the null hypothesis of at most one cointegration vector. The first column (L-Max) gives the estimated Johansen likelihood value in each case. The second and fourth columns give the 10% and 5% critical values taken from Osterwald-Lenum (1992, Table 1.1). The third and fifth columns give the small-sample-adjusted critical values. The adjustment factor is calculated as $T/(T-nk)$, where T is the number of observations (28), n is the number of variables including the intercept and drought dummy variable (7), and k is the number of lags (1). When the dummy is included (upper panel), the adjustment factor is 1.33; when it is excluded, this becomes 1.27. See Cheung and Lai (1993) for discussion of the adjustment factor.

TABLE 3: Long Run Parameter Estimates for Côte d'Ivoire. Dependent Variable is $\log(\text{REER})$.

	<i>OPEN1</i>	<i>OPEN2</i>	<i>OPEN3</i>	<i>OLS-ECM</i>	<i>IV-ECM</i>
<i>Constant</i>	3.61 (16.71)	4.29 (22.01)	4.30 (12.22)	1.72 (2.22)	1.35 (1.42)
<i>log(TOT)</i>	0.40 (3.03)	0.16 (1.06)	0.15 (0.94)	0.80 (2.07)	0.75 (2.21)
<i>RESGDP</i>	-2.67 (-5.49)	-1.47 (-3.25)	-1.45 (-3.71)	-0.89 (-0.49)	-1.53 (-1.04)
<i>log(OPEN)</i>	-0.78 (-3.68)	-0.08 (-0.34)	-0.03 (-0.12)	-0.28 (-0.42)	-0.46 (-0.82)
<i>log(ISHARE)</i>	-0.27 (-5.83)	-0.31 (-4.63)	-0.30 (-5.15)	-0.47 (-3.24)	-0.43 (-3.56)
<i>D₈₃₋₈₅</i>	-0.22 (-3.01)	-0.30 (-3.43)	-0.30 (-3.49)	-0.52 (-2.35)	-0.44 (-2.51)
<i>R²-Bar</i>	0.72	0.56	0.56	0.42	0.36
<i>Q</i>	14.32 (0.05)	13.80 (0.05)	14.21 (0.05)	7.16 (0.31)	4.68 (0.59)
<i>DW</i>	1.16	1.14	1.15	2.22	2.15
<i>DF</i>	-3.55	-3.31	-3.31		
<i>ADF</i>	-3.54	-3.84	-3.89		
<i>PP</i>	-3.61	-3.30	-3.29		

NOTES: The numbers in parentheses are t-ratios (note that these have non-standard distributions even asymptotically in columns 1-3). The static cointegration regressions in columns 1-3 use the three alternative openness variables discussed in Appendix 2. The last column reports the long-run parameters of the unrestricted ECM (equation (11) in the text; equivalent to the unrestricted ADL), using *OPEN1* as the openness variable. The long-run parameters and associated standard errors are obtained by estimating the Bewley transform of the ECM; see Banerjee, et al (1993) for details. The full set of parameters for this regression appear in column 1 of Table 4.

TABLE 4: ECM Parameter Estimates for Côte d'Ivoire. Dependent Variable is log(REER).

	<i>2-step ECM</i>		<i>Unrestricted ECM</i>	
	<i>OLS</i>	<i>IV</i>	<i>OLS</i>	<i>IV</i>
<i>Constant</i>	3.61 (16.71)	3.53 (15.68)	1.72 (2.22)	1.35 (1.42)
<i>Adjustment Speed</i>				
<i>log(REER_{t-1}) or Error_{t-1}</i>	-0.30 (-1.85)	-0.39 (-2.09)	-0.45 (-2.32)	-0.37 (-1.63)
<i>Long-Run Parameters</i>				
<i>log(TOT_{t-1})</i>	0.40 (3.03)	0.49 (3.29)	0.80 (2.07)	0.75 (2.21)
<i>RESGDP_{t-1}</i>	-2.67 (-5.49)	-2.81 (-5.58)	-0.89 (-0.49)	-1.53 (-1.04)
<i>log(OPEN_{t-1})</i>	-0.78 (-3.68)	-0.81 (-3.71)	-0.28 (-0.42)	-0.46 (-0.82)
<i>log(ISHARE_{t-1})</i>	-0.27 (-5.83)	-0.30 (-5.27)	-0.47 (-3.24)	-0.43 (-3.56)
<i>D_{83-85 t-1}</i>	-0.22 (-3.01)	-0.22 (-3.03)	-0.52 (-2.35)	-0.44 (-2.51)
<i>Short-Run Parameters</i>				
<i>Δlog(TOT_t)</i>	0.38 (2.86)	0.43 (2.97)	0.37 (1.78)	0.33 (1.44)
<i>ΔRESGDP_t</i>	-1.47 (-3.29)	-1.86 (-3.72)	-0.95 (-1.27)	-0.76 (-0.90)
<i>Δlog(OPEN_t)</i>	-0.38 (-1.99)	-0.49 (-2.59)	-0.29 (-0.95)	-0.28 (-0.87)
<i>Δlog(ISHARE_t)</i>	-0.10 (-1.72)	-0.10 (-1.40)	-0.18 (-2.37)	-0.11 (-0.96)
<i>Δlog(PFOR_{t-1})</i>	-0.30 (2.39)	-0.14 (-1.06)	-0.29 (-0.97)	-0.14 (-0.58)
<i>ΔD₈₃₋₈₅</i>	-0.05 (-1.04)	-0.05 (1.01)	-0.07 (-0.97)	-0.04 (-0.43)
<i>Q</i>	14.32 (0.05)	7.17 (0.31)	7.16 (0.31)	4.68 (0.59)
<i>R²-Bar</i>	0.49	0.74	0.42	0.36
<i>DW</i>	1.11	1.12	2.22	2.15

NOTES: The numbers in parentheses are t-ratios. The period of estimation is 1965-93. In columns 1 and 3, the long-run parameters and associated standard errors are obtained by estimating the Bewley transform of the ECM. In columns 1 and 2 we use the lagged residual from the static regression as the error-correction term. Columns 2 and 2 are instrumental variable estimates, using two lags of all right-side-variables as instruments for ISHARE.

TABLE 5: Observed and Equilibrium RER Indexes for Côte d'Ivoire – 1980 to 1993

<i>Year</i>	<i>Observed</i>	<i>Equilibrium RER</i>				<i>Overvaluation</i>
		<i>Fitted</i>	<i>5-year MA</i>	<i>B-N</i>	<i>"Sustainable"</i>	
1980	139	130	137	136	92	34
1981	121	121	120	124	94	22
1982	109	109	112	116	99	9
1983	104	104	108	121	107	-3
1984	100	100	103	121	131	-31
1985	100	100	103	104	112	-12
1986	126	116	128	115	118	6
1987	149	149	149	121	102	31
1988	149	149	149	132	97	35
1989	143	143	144	186	108	24
1990	152	152	149	185	121	20
1991	151	151	145	165	110	27
1992	164	164	153	168	108	34
1993	166	166	154	156	118	29

NOTES: The observed RER is the one used in the econometric analysis. The long-run parameter vector is taken from the static regression in column 1 of Table 3. "Fitted" values are obtained directly from that regression; "5-year MA" refers to five-year moving averages for all fundamentals; "B-N" refers to Beveridge-Nelson decompositions of all fundamentals; and the "sustainable" RER is defined as the fitted RER with all fundamentals replaced by counterfactual sustainable values, as determined in Appendix 3. Overvaluation is defined as $100 \times (\text{observed RER} - \text{sustainable RER}) / (\text{sustainable RER})$.

TABLE 6: Error Correction Model Parameter Estimates for Burkina Faso. Dependent Variable is $\Delta \log(REER_t)$

	<i>Unrestricted</i>		<i>Restricted</i>	
	<i>w/ Trend</i>	<i>w/o Trend</i>	<i>w/ Trend</i>	<i>w/o Trend</i>
<i>Constant</i>	0.92 (0.64)	1.21 (1.16)	1.59 (1.36)	2.78 (2.31)
<i>Trend</i>	0.01 (0.30)		0.01 (1.06)	
<i>Adjustment Speed</i>				
$\log(REER_{t-1})$	-0.50 (-2.76)	-0.51 (-2.87)	-0.54 (-2.81)	-0.60 (-3.20)
<i>Long-Run Parameters</i>				
$\log(TOT_{t-1})$	0.79 (1.20)	0.81 (1.28)	0.45 (1.37)	0.03 (0.13)
$\log(OPEN_{t-1})$	-1.02 (-0.92)	-0.78 (-1.15)	-0.78 (-1.37)	-0.06 (-0.20)
$RESGDP_{t-1}$	-7.69 (-1.62)	-6.87 (-1.97)	-5.69 (-2.15)	-2.20 (-1.88)
$\log(PFOR_{t-1})$	0.10 (0.48)	0.17 (1.28)		
<i>Short-Run Parameters</i>				
$\Delta \log(TOT_t)$	0.17 (0.74)	0.17 (0.74)		
$\Delta \log(OPEN_t)$	-0.13 (-0.42)	-0.08 (-0.32)		
$\Delta RESGDP_t$	-3.20 (-2.75)	-2.99 (-3.33)	-4.42 (-2.66)	-2.24 (-5.32)
$\Delta \log(PFOR_t)$	-0.30 (-1.31)	-0.30 (-1.42)		
$R^2\text{-Bar}$	0.73	0.75	0.72	0.72
Q	8.76 (0.12)	9.51 (0.09)	7.44 (0.19)	3.99 (0.55)
DW	2.24	2.20	1.99	2.01

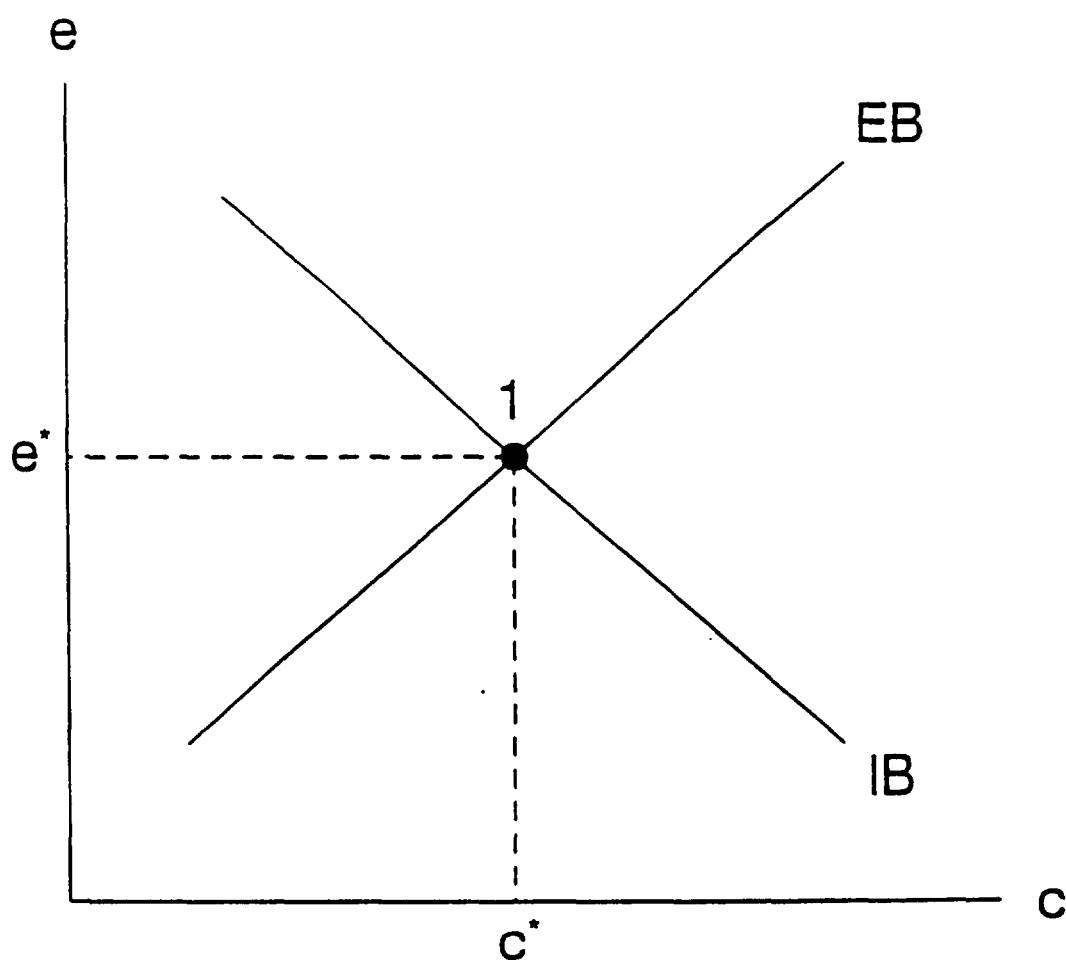
NOTES: Numbers in parentheses are t-ratios. The period of estimation is 1970-93. The unrestricted ECM corresponds to equation (11) in the text. The long-run parameters and associated standard errors are obtained by estimating the Bewley transform of the ECM.

TABLE 7: Observed and Equilibrium RER Indexes for Burkina Faso – 1980 to 1993

<i>Year</i>	<i>Observed</i>	<i>Equilibrium RER</i>				<i>Overvaluation</i>
		<i>Fitted</i>	<i>Trend</i>	<i>5-year MA</i>	<i>“Sustainable”</i>	
1980	115	92	106	93	87	31
1981	102	92	105	93	90	14
1982	104	94	105	91	100	4
1983	99	93	105	92	122	-18
1984	96	83	104	95	138	-16
1985	100	99	104	95	119	-6
1986	102	106	103	96	109	-2
1987	99	95	103	100	101	-4
1988	99	99	103	98	103	-16
1989	95	99	102	97	114	-11
1990	95	90	102	99	107	-13
1991	93	102	102	100	108	-12
1992	92	104	101	100	104	-12
1993	91	103	101	103	103	-27

NOTES: The observed RER is the one used in the econometric analysis. The fitted RER is the one estimated from the cointegration regression (Table 6). “Trend” refers to fitted linear trend for the RER. “5-year MA” refers to 5-year moving averages. The sustainable RER is the fitted RER where the fundamentals (i.e. RESGDP and OPEN) have been replaced by their sustainable counterparts as outlined in Appendix 3. Overvaluation is defined as $100 \times (\text{observed RER} - \text{sustainable RER}) / (\text{sustainable RER})$.

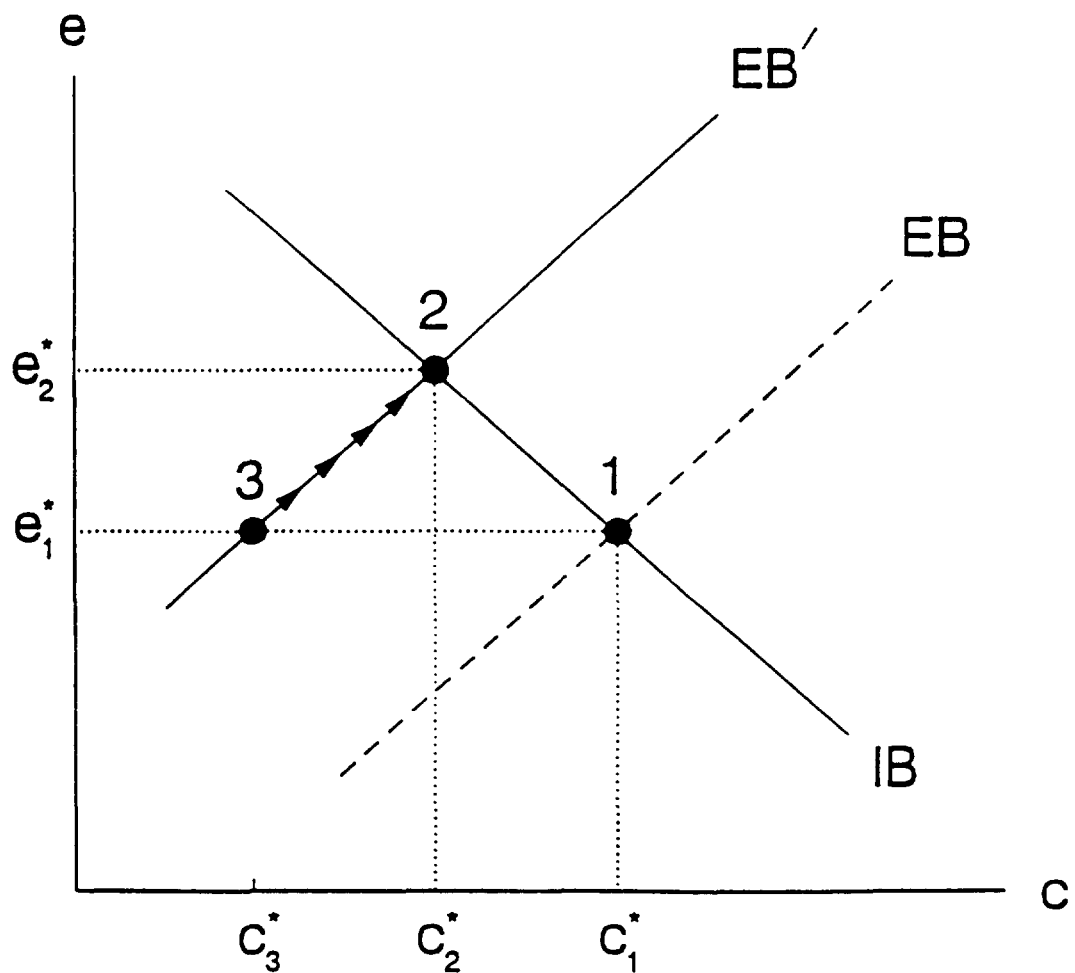
Figure 1
Internal and external balance



The EB schedule is drawn for steady-state values of the service account and transactions costs.
A rise in e is a real depreciation.

Figure 2

*Adjustment to an increase in r_w
(under a binding credit constraint)*



A rise in r_w shifts EB downwards to EB' . With flexible wages and prices, adjustment to the new long-run equilibrium at point 2 is immediate. With nominal rigidities, the economy jumps to point 3 and then converges gradually to point 2 along EB' .

FIGURE 3: VARIANCE RATIO TESTS FOR COTE D'IVOIRE

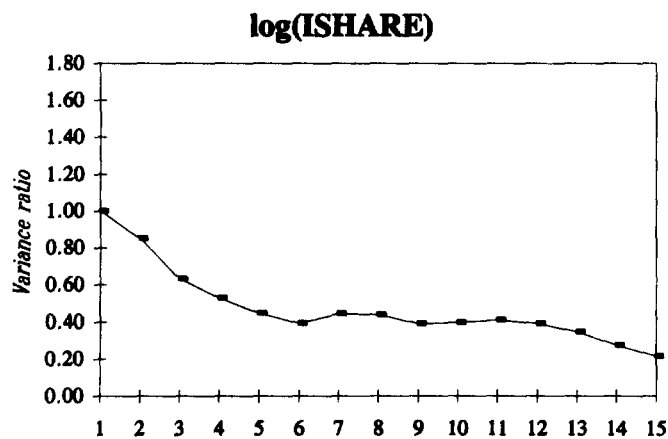
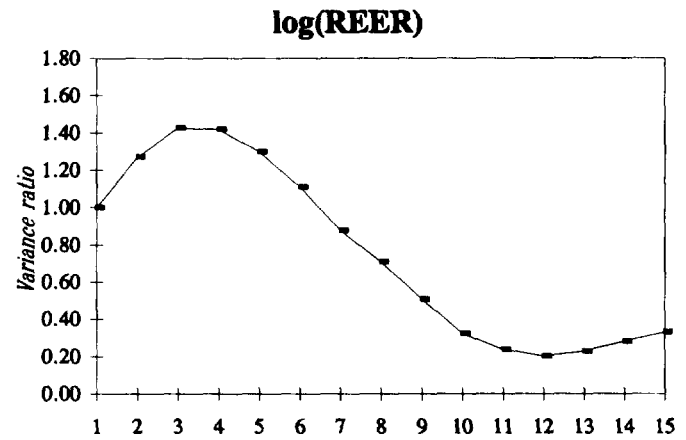
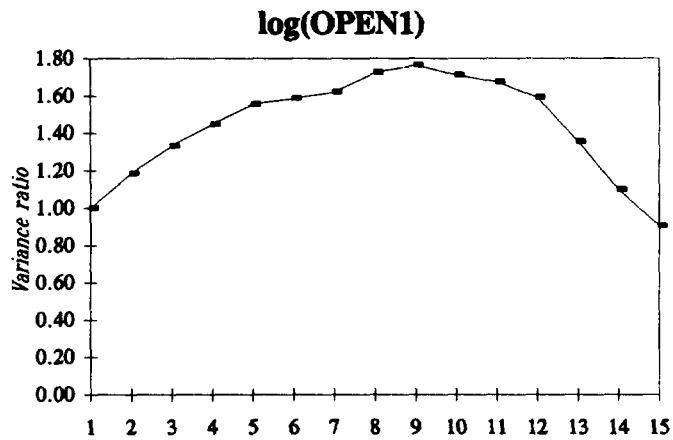
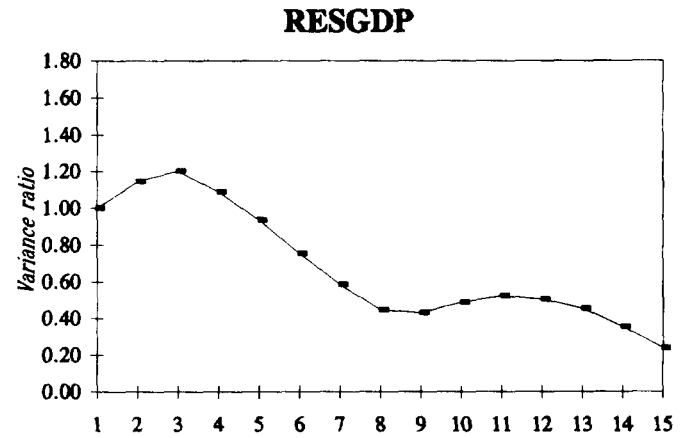
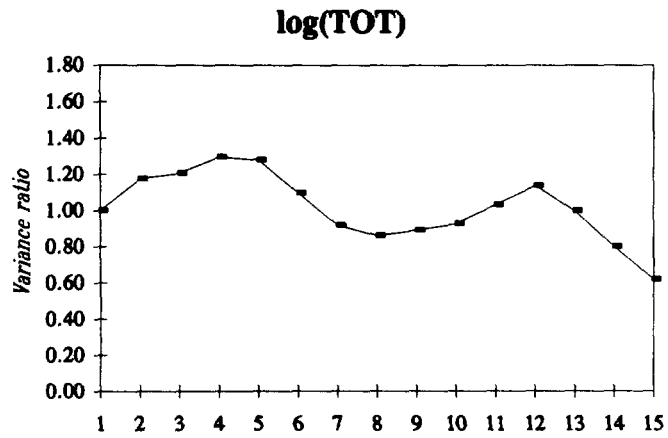
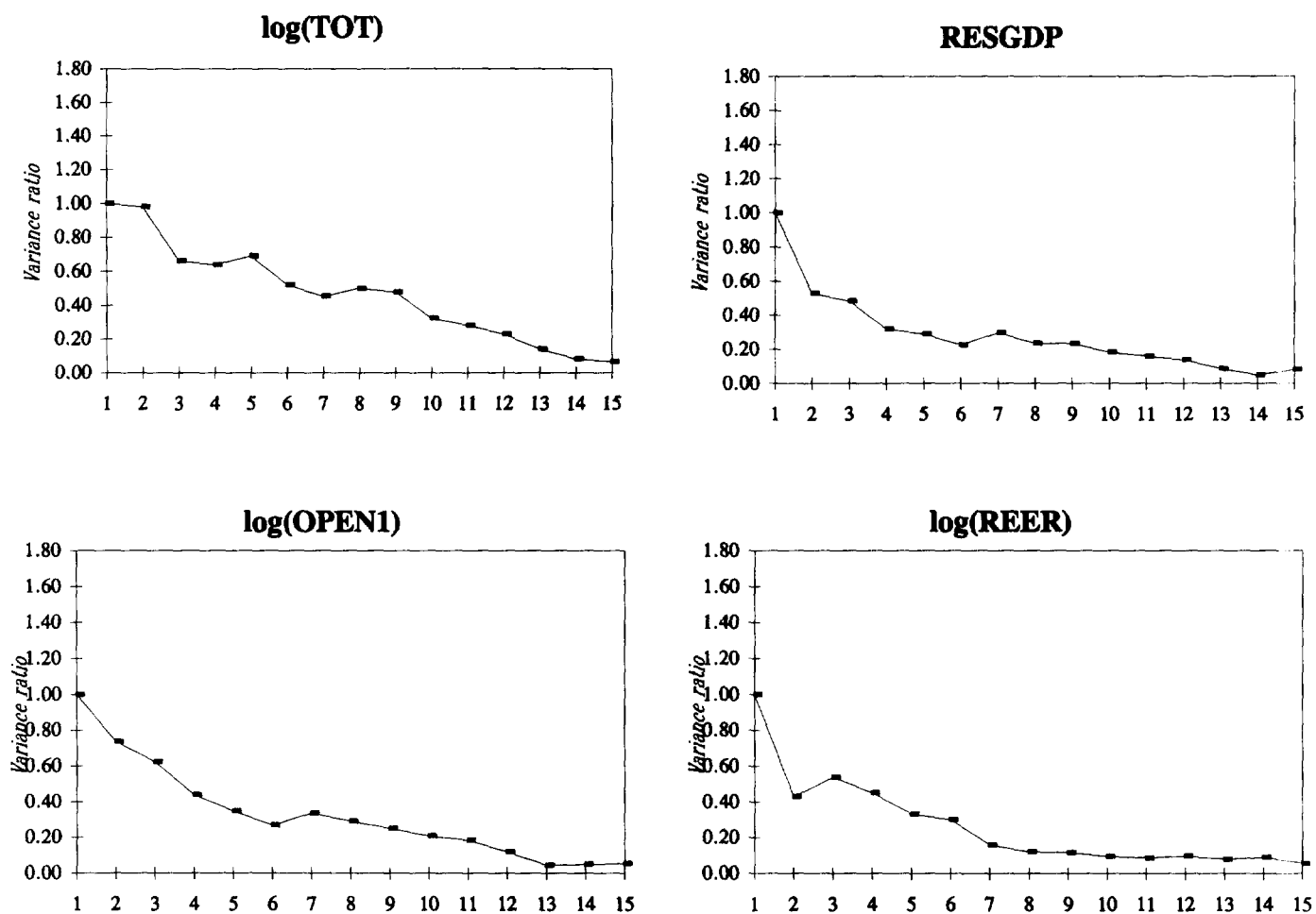


FIGURE 4: VARIANCE RATIO TESTS FOR BURKINA FASO



References

- Banerjee, A., J. Dolado, J. W. Galbraith and D. F. Hendry** (1993), *Co-Integration, Error-Correction, and the Econometric Analysis of Non-Stationary Data*, Oxford University Press, U.K.
- Bernanke, B.** (1986), "Alternative Explanations of the Money-Income Correlation", *Carnegie-Rochester Conference Series in Public Policy* 25, 49-99.
- Beveridge, S. and C. R. Nelson** (1981), "A New Approach to Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to Measurement of the Business Cycle", *Journal of Monetary Economics* 7, 151-174.
- Boswijk, H.** (1995), "Efficient Inference on Cointegration Parameters in Structural Error Correction Models", *Journal of Econometrics* 69, 113-158.
- Cheung, Y-W. and K. Lai** (1993), "Finite Sample Sizes of Johansen's Likelihood Ratio Tests for Cointegration", *Oxford Bulletin of Economics and Statistics* 55, 313-28.
- Cochrane, J. H.** (1988), "How Big Is the random Walk in GNP?" *Journal of Political Economy* 96, 893-920.
- Corden, W. M.** (1989), "Macroeconomic Adjustment in Developing Countries", *World Bank Research Observer* 4, 51-64.
- Devarajan, S.** (1997), "How Overvalued was the CFA? Estimates of Real Exchange Rate Misalignment with a Simple General Equilibrium Model", Mimeo, The World Bank.
- Devarajan, S. and L. Hinkle** (1995), "The CFA Franc Parity Change: An Opportunity to Restore Growth and Reduce Poverty," Mimeo, The World Bank.
- Devarajan, S., J. Lewis and S. Robinson** (1993), "External Shocks, Purchasing Power Parity, and the Equilibrium Real Exchange Rate", *World Bank Economic Review* 7(1): 45-63.
- Dollar, D.** (1992), "Outward-Oriented Developing Economies Really Do Grow More Rapidly", *Economic Development and Cultural Change* 40(3), April: 545-66.
- Dornbusch, R.** (1984), *Open Economy Macroeconomics*, New York: Basic Books.
- Edwards, S.** (1989), *Real Exchange Rates, Devaluation and Adjustment: Exchange Rate Policy in Developing Countries*, MIT Press, Cambridge, Massachusetts.
- Elbadawi, I.** (1994), "Estimating Long-run Equilibrium Real Exchange Rates," in *Estimating Equilibrium Exchange Rates*, J. Williamson, ed, Institute for International Economics, Washington, DC.
- Elbadawi, I. and R. Soto** (1994), "Capital Flows and Equilibrium Real Exchange Rates in Chile," *Policy Research Working Paper* 1306, The World Bank, Washington DC.
- Elbadawi, I. and R. Soto** (1995), "Real Exchange Rate and Macroeconomic Adjustment in Sub-Saharan Africa and Other Developing Countries," (forthcoming) in Elbadawi and Soto, eds, *Foreign Exchange Market and Exchange Rate Policies in Sub-Saharan Africa*, a special issue of *Journal of African Economies*.

- Engle, R. and C. Granger** (1987), "Co-Integration and Error-Correction: Representation, Estimation, and Testing", *Econometrica* 55, 251-76.
- Engle, R. F., D. F. Hendry and J-F Richard** (1983), "Exogeneity", *Econometrica* 51, 277-304.
- Ericsson, N. R.** (1995) "Conditional and Structural Error Correction Models", *Journal of Econometrics* 69, 159-71.
- Ericsson, N. R.** (1992) "Cointegration, Exogeneity, and Policy Analysis: An Overview", *Journal of Policy Modeling* 14, 251-80.
- Ericsson, N. R., J. Campos and H-A. Tran** (1991), "PC-Give and David Hendry's Econometric Methodology", *International Finance Discussion Papers* No. 406, Board of Governors of the Federal Reserve System, Washington DC.
- Granger, C. W. J.** (1981) "Some Properties of Time Series Data and Their Use in Econometric Model Specification", *Journal of Econometrics* 16, 213-28.
- Hamilton, J.** (1994), *Time Series Analysis*, Princeton University Press, N.J.
- Hargreaves, C.** (1994), "A Review of Methods of Estimating Cointegrating Relationships", Chapter 4 in C. Hargreaves, ed, *Nonstationary Time Series Analysis and Cointegration*, Oxford: Oxford University Press.
- Hendry, D. F.** (1995), *Dynamic Econometrics*, Oxford: Oxford University Press.
- Hendry, D., F., A. R. Pagan and J. D. Sargan** (1984), "Dynamic Specification", Chapter 18 in Z. Griliches and Michael D. Intriligator, eds, *Handbook of Econometrics, Vol 2*, Amsterdam: North-Holland: 1023-92.
- Hinkle, L. E. and F. Nsengiyumva** (1997), "The Relationship Between the External and Internal Real Exchange Rates: Competitiveness, Productivity, and the Terms of Trade," Mimeo, The World Bank.
- Johansen, S.** (1988), "Statistical Analysis of Cointegration Vectors", *Journal of Economic Dynamics and Control*, 12, 231-54.
- Johansen, S.** (1992), "Cointegration in Partial Systems and Efficiency of Single-Equation Analysis," *Journal of Econometrics* 52, 389-402.
- Johansen, S. and K. Juselius** (1994), "Identification of the Long-Run and the Short-Run Structure: An Application to the ISLM Model", *Journal of Econometrics* 63, 7-36.
- Kaminsky, G.** (1987), "The Real Exchange Rate in the Short Run and in the Long Run", Discussion Paper, Department of Economics, University of California at San Diego, December.
- Khan, M. S. and J. S. Lizondo** (1987), "Devaluation, Fiscal Deficits, and the Real Exchange Rate in Developing Countries", *World Bank Economic Review* 1, 357-74.
- M'Bet, A. and N. Madeleine** (1994), "External Shocks, Macroeconomic Adjustment and Behavior of the CFA Economies Under a Flexible CFA Pegging Scenario: The Cases of Côte d'Ivoire and Burkina Faso," Revised Report Presented at the AERC Workshop, Nairobi, Kenya.

- Monfort, A. and R. Rabemananjara** (1990), "From a VAR Model to a Structural Model: With an Application to the Wage-Price Spiral", *Journal of Applied Econometrics* 5, 203-27.
- Montiel, P.** (1997), "The Theory of the Long-Run Equilibrium Real Exchange Rate," Mimeo, The World Bank.
- Phillips, P. C. B.** (1995), "Fully Modified Least Squares and Vector Autoregression", *Econometrica* 63, 1023-78.
- Phillips, P. C. B. and Y. Chang** (1995), "Time Series Regression with Mixtures of Integrated Processes", *Econometric Theory* 11.
- Rodriguez, C. A.** (1994), "The External Effects of Public Sector Deficits", Chapter 2 in W. Easterly, C. A. Rodriguez, and K. Schmidt-Hebbel, eds, *Public Sector Deficits and Macroeconomic Performance*, Oxford: Oxford University Press.
- Rodrik, D.** (1993), "Trade Liberalization in Disinflation", CEPR Discussion Paper No. 832, London: Centre for Economic Policy Research, August.
- Serven, L.** (1996), "Does Public Capital Crowd Out Private Capital? Evidence from India," Policy Research Working Paper 1613, World Bank, Washington DC.
- Urbain, J.-P.** (1992), "On Weak Exogeneity in Error-Correction Models", *Oxford Bulletin of Economics and Statistics* 54, 187-207.
- Williamson, J., ed.** (1994), *Estimating Equilibrium Exchange Rates*, Institute for International Economics, Washington, DC.
- World Bank** (1989), *Sub-Saharan Africa: From Crisis to Sustainable Growth: A Long-Term Perspective Study*, Washington, D.C.

Appendix 1: Conditioning and weak exogeneity

Weak exogeneity is a property of the joint distribution of the real exchange rate and the fundamentals. In this appendix we introduce the concept of conditional and marginal models and explore the relationship between the single-equation model (11) and the full distribution of the $(nx1)$ vector $[ln e_t, F_t', z_t']'$, conditional on its own past (see also Ericsson (1992)). With reasonable generality we can describe this distribution as the p -th-order Gaussian vector autoregression (VAR)

$$x_t = \sum_{j=1}^p \Pi_j x_{t-j} + \varepsilon_t, \quad \varepsilon_t \sim IN(0, \sigma^2), \quad (A1)$$

where the Π_j are (nxn) matrices of reduced-form coefficients and is the nxn symmetric and positive definite matrix of contemporaneous covariances between the innovations ε_{it} . Equation (A1) can be written equivalently as

$$\Delta x_t = \Gamma x_{t-1} + \sum_{j=1}^p A_j \Delta x_{t-j} + \varepsilon_t, \quad (A2)$$

where $\Gamma = [(\sum_{j=1}^p \Pi_j) - I]$ and $A_j = -\sum_{i=j+1}^p \Pi_i$. The first row of (A2) is a *reduced-form* error-correction model for Δlne_t ; it is similar to (11) but excludes contemporaneous values of F and z . To obtain the distribution of lne_t conditional on lagged x_t and contemporaneous F and z , we first partition the vector x_t into $x_t = [ln e_t, w_t']'$, where $w_t = [F_t', z_t']'$ is the vector of macroeconomic determinants of the real exchange rate. Without loss of generality, we can then factorize the joint distribution represented by (A2) into the distribution of Δlne_t conditional on contemporaneous w_t 's (and lagged x_t 's) and the associated marginal distribution of the w_t 's (given lagged x_t 's). Under normality of ε_t , the conditional and marginal models take the form

$$\Delta lne_t = \Sigma_{12}(\Sigma_{22})^{-1} \Delta w_t + (\Gamma_1 - \Sigma_{12}(\Sigma_{22})^{-1})x_{t-1} + \sum_{j=1}^p (A_{1j} - \Sigma_{12}(\Sigma_{22})^{-1} A_{2j}) \Delta x_{t-j} + \xi_t, \quad (A3a)$$

$$\Delta w_t = \Gamma_2 x_{t-1} + \sum_{j=1}^p A_2 \Delta x_{t-j} + \varepsilon_{2t}, \quad (A3b)$$

where the numerical subscripts refer to the blocks of appropriately partitioned matrices. By construction, the disturbance term in (A3a), $\xi_t = \Sigma_{11} - \Sigma_{12}(\Sigma_{22})^{-1} \Sigma_{21}$ is uncorrelated with all of the

variables on the right-hand side of that equation. Equation (A3) follows the standard regression relationship between two jointly normal scalar random variables y and z , i.e., the conditional distribution of y_t is given by $y_t = \mu_1 + (\sigma_{12}/\sigma_{22})(z_t - \mu_2) + v_t$, where the μ_i 's are means and the σ_{ij} 's are covariances; and the disturbance v_t has the properties $E(v_t | z_t) = 0$ and $Var(v_t | z_t) = \sigma_{11} - (\sigma_{12})^2/\sigma_{22}$. That this representation is simply a re-parameterization of (A2) can be confirmed by pre-multiplying (A2) by the $n \times n$ nonsingular matrix

$$B = \begin{bmatrix} I & \Sigma_{12}(\Sigma_{22})^{-1} \\ 0 & I_{m-1} \end{bmatrix},$$

which results in (A3).

Equation (A3a) is a single-equation conditional error-correction model whose general form mimics that of equation (11). Although it is often assumed in writing an equation like (11) that the disturbance is uncorrelated with the right-hand side variables, this is true *by construction* for equation (A3a). To the degree that the parameterizations differ, therefore, OLS estimation of (11) will tend to uncover the parameters of (A3a) (in which orthogonality holds by construction), yielding inconsistent estimates of the parameters of (11). Moreover, even if the parameters of (11) can be recovered from those of (A3a), the latter are potentially complicated functions of the underlying VAR parameters. There may therefore be cross-equation restrictions linking these parameters to those of the marginal model (A3b). In such a case efficient estimation of the conditional model requires that these restrictions be imposed; and failure to impose them may produce inconsistent standard errors, invalidating inference.

These considerations motivate a search for conditions under which estimation and inference regarding particular parameters of (11) can proceed successfully in the conditional model alone (i.e., without analyzing the full system). In such cases the sub-vector w_t is said to be *weakly exogenous* for the parameters of interest (Engle, Hendry and Richard (1983)). In the context of the above discussion, weak exogeneity requires (a) that the parameters of interest can be directly recovered from those of the conditional model; and (b) that there be no cross-equation restrictions linking these parameters to those of the marginal model.

Appendix 2: Data Description

The data were taken from three sources: (1) IMF, *International Financial Statistics*, (2) UNCTAD, and (3) the World Bank's Unified Survey. The variables were constructed as follows:

Real Exchange Rate (RER). Ratio of the domestic consumer price index (CPI) to the trade-weighted foreign wholesale price index (WPI), multiplied by the trade-weighted nominal exchange rate (NER): $RER = (CPI/WPI) * NER$.

Terms of Trade (TOT). Ratio of export price index (P_X) to import price index (P_M) (expressed in dollars, taken from UNCTAD): $TOT = P_X/P_M$.

Openness (OPEN). OPEN1 is the import to GDP ratio (IMPGDP), and is defined as the value of imports at current prices (IMPCP) over GDP at current prices (GDPKP): $OPEN1 = IMPCP / GDPKP$. OPEN2 is the ratio of the value of imports at constant prices (IMPKP) plus exports at constant prices (EXPKP) to GDP at constant prices (GDPKP): $OPEN2 = (IMPKP + EXPKP) / GDPKP$. OPEN3 is the ratio of imports at constant prices to domestic absorption at constant prices: $OPEN3 = IMPKP / (GDPKP - (EXPKP - IMPKP))$.

Resource Balance to GDP Ratio (RESGDP). Value of exports at constant prices (EXPKP) minus value of imports at constant prices (IMPKP), divided by GDP at constant prices (GDPKP). EXPKP has been adjusted by the domestic terms of trade (TOTD) which are defined as the ratio of export to import deflator. Thus $RESGDP = (EXPKP * TOTD - IMPKP) / GDPKP$.

Investment Share (ISHARE). Ratio of gross investment at constant prices (IGROSS) to the sum of private consumption (PCONK), government consumption (GCONK), and gross investment, all at constant prices: $ISHARE = IGROSS / (PCONK + GCONK + IGROSSK)$.

Foreign Price Level (PFOR). Domestic consumer price index (CPI) divided by the real effective exchange rate (RER): $PFOR = CPI / RER$.

Appendix 3: Sustainable Fundamentals

A3.1 Time-series measures: TOT and LPFOR

Both Burkina Faso and Côte d'Ivoire are very small economies by world standards and are therefore price takers in the markets for both their exports and imports. Moreover, the nominal exchange rate for the CFA francs was fixed throughout the 1970-93 sample period and could not be changed by individual CFA countries. The terms of trade (TOT) and the foreign price level converted to CFA francs (LPFOR) are therefore exogenous variables. While these variables fluctuate substantially from year to year, we have no basis on which to question the sustainability" of their longer-run movements. We therefore use 5-year centered moving averages as the sustainable values of these variables (extrapolating out of sample using the first and last-year values). We also generate alternative sustainable values for Burkina Faso and Côte d'Ivoire using sample means and Beveridge-Nelson decompositions, respectively.

A3.2 Counterfactual simulations: RESGDP

RESGDP is the ratio of the resource balance to GDP, both in constant prices. Since Burkina Faso relied heavily on concessional aid flows in 1970-93, determining a sustainable resource balance is essentially a problem of determining sustainable levels of financial inflows. These inflows can be divided into net factor income, net transfers, and net capital flows. We used 5-year moving averages for the first two (interest payments were small and changed very slowly over the sample, so we ignored the feedback from borrowings to interest payments). We then divided net capital flows into its dominant component – net long-term concessional borrowing – and “other” flows (net direct investment, net portfolio investment, net short term borrowing, net errors and omissions), using 5-year moving averages for the latter. The government of Burkina Faso attempted to maximize net concessional borrowing during the sample period, so this component was ultimately determined by the foreign donors. To smooth out year-to-year fluctuations in net concessional borrowing, we used the smaller of the 5-year moving average of the actuals or 3.5% of GDP (the highest level reached except in drought years). The sustainable resource balance is then the sum of these sustainable components. Note that the Bank's debt stock and flow data are not consistent with the national accounts and balance of payments data for Burkina Faso and Côte d'Ivoire. Since the balance of payments and national accounts data are consistent with each other and essential for the analysis, we used balance of payments data when there were conflicts between these and Bank's debt data.

The Côte d'Ivoire case is both more complicated and more representative of the problems likely to emerge in developing country applications. Côte d'Ivoire avoided balance of payments and debt problems in the 1970s. We therefore treated actual flows as essentially sustainable during the 1965-79 period, using 5-year moving averages to smooth out temporary fluctuations. After 1980, it was unable to meet its debt service payments. Moving averages therefore seem unlikely to capture sustainable movements in net borrowing and interest payments after 1980, and we cannot ignore the feedback from higher debt levels to higher interest payments. For 1980-93 we proceed as follows.

To proxy the sustainable level of borrowing, we used zero net repayments and net disbursements after 1979 (i.e., no change in the debt stock other than through write-downs). Côte d'Ivoire's debt ratio jumped from 47% in 1979 to 62% in 1980, then climbed to 115% in 1985 after which the country defaulted. The Maastricht Treaty, after which the fiscal guidelines for the West African Monetary Union are modelled, sets 60% of GDP as the maximum desirable debt level for the EU countries. A developing country might be able to target a somewhat higher debt level than 60% depending upon its rate of growth and its access to financing on concessional terms; so 1979 is by these criteria the last year of sustainable debt levels.

We calculate sustainable direct and portfolio investment as assumed percentages of total sustainable investment as determined below; together with the sustainable borrowing figures, these yield a sustainable level of total capital inflows.

To proxy sustainable interest payments, we use 4% of GDP. This represents a kind of compromise between a normative scenario in which interest payments are capped at 2.5% of GDP and a positive scenario (essentially feasibility calculation) that caps them at 5%. For comparison, the Maastricht debt ceiling, with an inflation rate of 3% and a real interest rate of 3% implies interest payments of 1.8% of GDP for the EU countries. Côte d'Ivoire was unable to sustain the service payments on its debt after interest payments reached 3.5 and 5.2% of GDP in 1981 and 82.

The sustainable resource deficit for 1980-1993 is then calculated as the sum of net transfers, net factor income, and net capital inflows, using 5-year moving averages of the actuals for transfers and factor income flows other than interest payments.

A3.3 Counterfactual simulations: ISHARE and OPEN1

ISHARE is the ratio of investment to GDP in constant prices; OPEN1 is the ratio of imports to absorption in current prices. The sustainability criterion we use for these variables is consistency

with a 3% long run growth rate of GDP per capita.

With population growth estimated at about 3% for both countries over the sample, GDP growth of 6% is required to achieve 3% growth in GDP per capita. Using ICORs of 4 for Côte d'Ivoire and 5 for Burkina Faso, this would in turn require investment ratios of about 25% and 30% of GDP, respectively. The 25% ratio is in line with those actually achieved during 1960s and 70s in Côte d'Ivoire; it is also the target that the World Bank has suggested as a guideline for Africa as a whole (World Bank (1989)). For Côte d'Ivoire, therefore, we use a moving average of the actual investment levels for 1965 to 1981, which were reasonably close to 25%, and 20% for 1982-93 when investment was depressed far below this level. For Burkina Faso, where the investment/GDP ratio is used only as an input to calculate the target import/absorption ratio (see below), we assume a sustainable investment ratio of 25%.

For both countries we assume that increases in the import to GDP ratio were required to deliver the import content of additional investment and also support a more liberal trade regime. We estimate an import content of investment of roughly 0.6 for both countries. To incorporate trade liberalization, we assume increases in the import ratio of 3% and 2%, respectively, for Côte d'Ivoire and Burkina Faso. The target import ratio is then estimated as the actual import ratio plus 3% of GDP plus 0.6 times the difference between the target investment ratio and the actual investment ratio. This target import/absorption ratio is used for the entire sample period as a more open trade policy would have been desirable throughout.

A3.4 A caveat

As the above discussion suggests, determining target values for particular countries requires considerable country specific knowledge and a number of assumptions based on partial information and analysis. These assumptions are open to question, and different ones – regarding either the key parameters or the underlying notion of sustainability – would yield different results. It may therefore be important in specific cases to consider alternative plausible assumptions and to compare the results of the various alternatives to those from using moving averages for the target variables.

Policy Research Working Paper Series

Title	Author	Date	Contact for paper
WPS1773 The Costs and Benefits of Regulation: Implications for Developing Countries	J. Luis Guasch Robert W. Hahn	June 1997	J. Troncoso 38606
WPS1774 The Demand for Base Money and the Sustainability of Public Debt	Valeriano F. Garcia	June 1997	J. Forgues 39774
WPS1775 Can High-Inflation Developing Countries Escape Absolute Poverty?	Martin Ravallion	June 1997	P. Sader 33902
WPS1776 From Prices to Incomes: Agricultural Subsidization Without Protection?	John Baffes Jacob Meerman	June 1997	P. Kokila 33716
WPS1777 Aid, Policies, and Growth	Craig Burnside David Dollar	June 1997	K. Labrie 31001
WPS1778 How Government Policies Affect the Relationship between Polish and World Wheat Prices	Szczepan Figiel Tom Scott Panos Varangis	June 1997	J. Jacobson 33710
WPS1779 Water Allocation Mechanisms: Principles and Examples	Ariel Dinar Mark W. Rosegrant Ruth Meinzen-Dick	June 1997	M. Rigaud 30344
WPS1780 High-Level Rent-Seeking and Corruption in African Regimes: Theory and Cases	Jacqueline Coolidge Susan Rose-Ackerman	June 1997	N. Busjeet 33997
WPS1781 Technology Accumulation and Diffusion: Is There a Regional Dimension?	Pier Carlo Padoan	June 1997	J. Ngaine 37947
WPS1782 Regional Integration and the Prices of Imports: An Empirical Investigation	L. Alan Winters Won Chang	June 1997	J. Ngaine 37947
WPS1783 Trade Policy Options for the Chilean Government: A Quantitative Evaluation	Glenn W. Harrison Thomas F. Rutherford David G. Tarr	June 1997	J. Ngaine 37947
WPS1784 Analyzing the Sustainability of Fiscal Deficits in Developing Countries	John T. Cuddington	June 1997	S. King-Watson 31047
WPS1785 The Causes of Government and the Consequences for Growth and Well-Being	Simon Commander Hamid R. Davoodi Una J. Lee	June 1997	E. Witte 85637
WPS1786 The Economics of Customs Unions in the Commonwealth of Independent States	Constantine Michalopoulos David Tarr	June 1997	M. Pateña 39515

Policy Research Working Paper Series

Title	Author	Date	Contact for paper
WPS1787 Trading Arrangements and Industrial Development	Diego Puga Anthony J. Venables	June 1997	J. Ngaine 37947
WPS1788 An Economic Analysis of Woodfuel Management in the Sahel: The Case of Chad	Kenneth M. Chomitz Charles Griffiths	June 1997	A. Maranon 39074
WPS1789 Competition Law in Bulgaria After Central Planning	Bernard Hoekman Dimeon Djankov	June 1997	J. Ngaine 37947
WPS1790 Interpreting the Coefficient of Schooling in the Human Capital Earnings Function	Barry R. Chiswick	June 1997	P. Singh 85631
WPS1791 Toward Better Regulation of Private Pension Funds	Hemant Shah	June 1997	N. Johl 38613
WPS1792 Tradeoffs from Hedging Oil Price Risk in Ecuador	Sudhakar Satyanarayan Eduardo Somensatto	June 1997	E. Somensatto 30128
WPS1793 Wage and Pension Pressure on the Polish Budget	Alain de Crombrughe	June 1997	M. Jandu 33103
WPS1794 Ownership Structure, Corporate Governance, and Corporate Performance: The Case of Chinese Stock Companies	Xiaonian Xu Yan Wan	July 1997	J. Chinsen 34022
WPS1795 What Educational Production Functions Really Show: A Positive Theory of Education Spending	Lant Pritchett Deon Filmer	July 1997	S. Fallon 38009
WPS1796 Cents and Sociability: Household Income and Social Capital in Rural Tanzania	Deepa Narayan Lant Pritchett	July 1997	S. Fallon 38009
WPS1797 Formal and Informal Regulation of Industrial Pollution: Comparative Evidence from Indonesia and the United States	Sheoli Pargal Hemamala Hettige Manjula Singh David Wheeler	July 1997	E. de Castro 89121
WPS1798 Poor Areas, Or Only Poor People?	Martin Ravallion Quentin Wodon	July 1997	P. Sader 33902
WPS1799 More for the Poor Is Less for the Poor: The Politics of Targeting	Jonath B. Gelbach Lant H. Pritchett	July 1997	S. Fallon 38009